A MANUAL

OF

LABORATORY PHYSICS

BY

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AND

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FIRST EDITION.

FIRST THOUSAND.

NEW YORK:
JOHN WILEY & SONS.
LONDON: CHAPMAN & HALL, LIMITED.
1901.

Elementary Lessons in Electricity and Magnetism, Silvanus Thompson; A Text-book of Physics, Watson; Physics for University Students, Carhart; General Physics, Hastings and Beach; Physics, Advanced Course, Barker; Theory of Physics, Ames.

Under "Apparatus Required" will be found an exact statement of the apparatus necessary for the particular experiment.

Under "Theory of Experiment" will be found set forth the theory involved in the special experiment under consideration. As most students come to the laboratory with very imperfectly formed ideas of physical theory, this portion of each experiment has been found especially useful, as it gives to the student a clear conception of the principles involved before he begins the actual practice of the experiment.

Under "Practical Directions" will be found just such directions as a Demonstrator would give to a student if standing beside him.

In addition, a tabulated example of the observations and results has been added to serve as a guide to the student. In cases where doubt might arise the calculations involved will also be found.

The examples have been taken mostly from the work done by students, and will serve to give an idea of the order of accuracy possible.

The "Blank to be filled in by Student" has been added to enable the student to keep a permanent record of his work. The results should be first returned to the Demonstrator in tabulated form and approved of before they are entered in the book.

Most of the manuscripts were prepared when Mr. Pitcher and I were fellow Demonstrators in the laboratories. On Mr. Pitcher's retirement from the University and my own retire-

ment as a teacher from the Physical Department, the work of publication was undertaken at the request of the Professors in charge. During the past year I have revised and completed the separate manuscripts, reducing them to the present uniform pattern. As a result of the method of treatment some apparent repetitions occur under "Theory of Experiment," but I have preferred to let these remain, so that each experiment stands in a sense complete by itself, thus permitting the order of work to be varied.

For much help and many suggestions in the original drafting of the manuscripts we are indebted to Prof. Callendar, to whom some of the manuscripts, especially those on the D'Arsonval galvanometer, are due.

Constant reference has been had to text-books of Practical Physics, especially to those of Glazebrook and Shaw (which work was originally used by us as a text-book), Stewart and Gee, Nichols, and Kohlrausch (Physical Measurements).

As most of the proof-sheets have been read only by myself, I doubt not that some inaccuracies still remain, though none I hope which can be considered of any consequence.

Especial thanks are due to Mr. H. T. Barnes, D.Sc., Lecturer in Physics, for valuable assistance in collecting materials for some of the tabulated examples.

H. M. TORY.

McGILL COLLEGE, February 9, 1901.

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LABORATORY PHYSICS.

SOUND.

I. TO DETERMINE THE VIBRATION FREQUENCY OF A TUNING-FORK BY MEANS OF A SONOMETER.

References.—Knott, p. 261; Hastings and Beach, p. 563; Carhart, pt. I. p. 186; Nichols and Franklin, vol. III. p. 160; Ames, p. 173; Anthony and Brackett, p. 165; Watson, p. 392; Barker, p. 231.

Apparatus Required. — A sonometer; a tuning-fork, provided with a resonator; a rubber hammer for exciting the fork.

Theory of Experiment.—If a string stretched under a tension T, so great that the action of gravity may be neglected in comparison with it, be made to vibrate by drawing it aside at one point and then suddenly freeing it, the disturbance will be transmitted along the string as a wave motion, the velocity of the wave being given by the equation

$$V=\sqrt{\frac{T}{m}}, \qquad (1)$$

where m is the mass of the string per unit length.

If l be the length of the vibrating portion of the string, and n the vibration frequency for the fundamental note, then

$$V=2ln,$$

and therefore

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In formula (2) all the laws of the transverse vibration of strings are included.

If the length of the vibrating portion of the string be adjusted till the emitted note is the same as that of a tuning-fork, the vibration frequency of the fork can be calculated from the formula.

Practical Directions.—We shall assume that the usual form of sonometer, provided with weights for altering the tension, and with a movable bridge for altering the length of the vibrating portion of the string, is used.

A piece of piano-wire of small diameter is generally suitable for the purpose of the experiment.

If the weight m of the unit length of the wire be not given, the wire must be weighed before attaching it to the sonometer-box, and m calculated.

Fasten one end of the wire to the sonometer-box, and the other to the attachment for holding the weights.

Stretching the wire over the pulley, attach 20 to 30 pounds weight.

Excite the fork by a blow from the hammer.

Vibrate the wire by snapping it with the fingers at the middle point.

Continue the process, adjusting the length of the vibrating wire by means of the movable bridge, until the string and fork are in unison.

Care must be taken to adjust the string so that the fundamental note is in unison with the fork. To make quite sure of this, sound the first harmonic by vibrating the string one-quarter of its length from the end and touching it lightly at the middle with the finger. This being the first harmonic,

and an octave above the fundamental of the string, will enable the ear to determine at once whether the fundamental is being tuned to the fork or not.

If it be found that less than one-third of a meter of wire is used in the vibrating portion the tension should be increased so that the length of the string can be increased, otherwise the string will vibrate for such a short period that it will be almost impossible to make the comparison.

Measure in centimeters the length of the vibrating wire.

Read the stretching weight and reduce it to dynes.

Calculate n from formula (2).

Repeat the observations three times, altering the weights each time.

Example.—Enter results thus:

$m = .0424$

Observation.	l	Weight.	T in Dynes.	n
1st........	28.4	5 lbs.	2224908	128
2d..........	40.0	10 "	4449816	127.8
3d.........	49.0	15 "	6674724	127.7
4th.........	56.5	20 "	8899632	128 4
Mean value of n..........................				127.9

$T = 5 \times 453.6 \times 981$ Dynes, 1st observation.

$n = \frac{1}{2 \times 28.4}\sqrt{\frac{2224908}{.0424}} = 128$, 1st "

Blank to be filled in by student.

$m =$

Observation.	l	Weight.	T in Dynes.	n
Mean value of n				

$T =$

$n =$

2. TO DETERMINE THE VELOCITY OF SOUND IN AIR BY MEANS OF A RESONANCE-TUBE.

References.—Hastings and Beach, p. 562; Ames, p. 184; Barker, p. 220; Watson, p. 367; Anthony and Brackett, p. 161; Knott, p. 290; Carhart, pt. I. p. 146.

Apparatus Required.—A resonance-tube; a tuning-fork; a rubber hammer; a thermometer.

Theory of Experiment.—If a tuning-fork be made to vibrate over the open end of a tube closed at one end and of suitable length, the tube will act as a resonator, reinforcing the vibrations of the fork. The length of tube required is such that the time it takes the vibration to travel to the closed end of the tube and back must be the same as the time of a half-vibration of the fork.

If v be the velocity of sound in air, l the length of the tube, n the number of vibrations per second of the fork, and t the time of a semi-vibration, then

$$t = \frac{1}{2n} = \frac{2l}{v},$$

or

$$v = 4nl. \qquad (1)$$

It is evident that the fork will be reinforced not only when the time corresponds to the first half-vibration, but will also be reinforced if the length of the tube be such that the time it takes the vibration to travel to the closed end and back be equal to the time of a complete vibration and a half or any odd number of semi-vibrations of the fork.

It is therefore evident that if the tube be such that its length can be adjusted, a series of points of maximum resonance can be found. Hence

$$v = \frac{4nl}{2x + 1}, \qquad (2)$$

where x is 0, 1, 2, 3, etc., corresponding to the first, third, fifth, etc., semi-vibration.

This formula is not strictly accurate. A correction for the open end of the tube is necessary. The correction is nearly equivalent to adding to the length, l, of the tube its radius.

The formula therefore becomes

$$v = \frac{4n(l + r)}{2x + 1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

r being the radius of the tube.

If now the temperature of the air in the tube be t, the velocity v, then the velocity at zero, v_0, is given by the equation

$$v_0 = \frac{v}{\sqrt{1 + .00366t}},$$

or

$$v_0 = \frac{4n(l + r)}{(2x + 1)\sqrt{1 + .00366t}}. \quad . \quad . \quad . \quad (4)$$

Practical Directions.—A simple form of instrument suitable for the experiment consists of two glass tubes arranged to slide up and down in a wooden frame, the tubes being connected at the lower ends by a rubber tube (Fig. 1).

When the apparatus is partially filled with water, by adjusting the relative positions of the tubes the water may be made to rise and fall in either as desired.

Adjust the larger tube until the water rises nearly to the top of the smaller one.

Hold the vibrating fork horizontally over the mouth of the smaller tube, and adjust by means of the larger the height of water in the smaller, until a point of maximum resonance is obtained.

As this point is not very sharply defined, several separate adjustments should be made, and the mean of the observa-

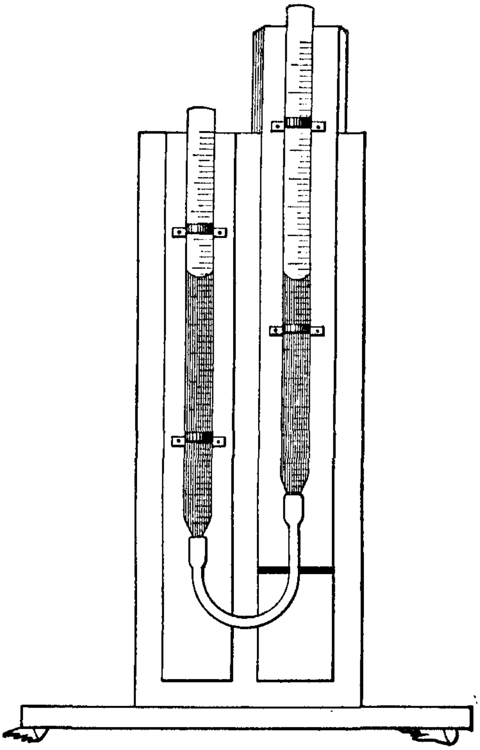

FIG. 1.

tions taken. The length, l, should be measured each time to 1 mm.

Take the temperature, t, of the air in resonance-tube.

Repeat the observations for third and fifth semi-vibrations if the tube is long enough.

A fork of 256 D. V. is very suitable for the experiment.

A suitable hammer for vibrating can be made by inserting a stiff wire into a rubber bottle-cork.

Example.—Enter the observations thus:

l, 1st Point.	t	l, 2d Point.	t	l, 3d Point.	t
31.5 32.1 31.7 31.9	15	98.4 99.0 99.1 98.3	15	Tube not long enough.	
Mean = 31.8		= 98.7		=	

$$v_0 = \frac{4 \times 256(31.8 + 1.5)}{\sqrt{1 + .003665 \times 15}} = 33200, \quad \text{1st Point.}$$

$$= \frac{4 \times 256(98.7 + 1.5)}{3\sqrt{1 + .00366 \times 15}} = 33300, \quad \text{2d Point.}$$

$$\text{Mean value of } v_0 = 33250 \text{ cm. per sec.}$$

Blank to be filled in by student.

l, 1st Point.	t	l, 2d Point.	t	l, 3d Point.	t
Mean =		=		=	

$v_0 =$ =

=   =

=   =

Mean value of v_0 =

3. TO DETERMINE THE FREQUENCY OF THE NOTE EMITTED BY AN ORGAN-PIPE, BY MEANS OF THE SIREN.

References.—Watson, p. 375; Anthony and Brackett, p. 165; Hastings and Beach, p. 542; Ames, p. 150; Knott, p. 273; Nichols and Franklin, vol. III. p. 150; Barker, p. 217.

Apparatus Required.—A siren with suitable speed-governor; an organ-pipe and tuning-fork of approximately the same frequency; a large gasometer and a pair of bellows for filling it; an experimental organ-bellows for furnishing the blast for the organ-pipe; a pressure-gauge; rubber connecting tubing; a supply of weights for loading bellows; a stop-watch.

Theory of Experiment.—If the air in an organ-pipe be excited by a blast of constant pressure, and a siren, having a speed of n revolutions per second while receiving its impulse through p holes per revolution, be brought either into unison with the note of the pipe or to differ from it by a known number of beats, b, per second, the frequency F of the organ-pipe can be determined.

For, if in unison with the siren, $F = pn$,
or if beating, $F = pn + b$.

If some means be employed by which the revolutions of the siren can be kept constant so that the beats can be counted, for a sufficient time, the above theory can be realized in practice.

Practical Directions.—Select an organ-pipe and connect it to the bellows.

Adjust the pressure of the blast by weights till the fundamental note is obtained.

Connect the foot-bellows to the gasometer and force it full of air.

Connect the gasometer to the siren and pressure-gauge.

See that the speed-counter of the siren engages in the worm carried by the spindle.

Set the organ-pipe and siren sounding, and weight the gasometer till the siren gives approximately the note of the organ-pipe without consuming more air than can easily be supplied by the bellows working constantly.

By adjusting the speed-governor, bring the frequency of the siren up or down as required, till the beats are fairly constant for 2000 or so revolutions and slow enough to be easily counted.

Count the total number of beats during 1000 or 2000 revolutions, timing the revolutions by the stop-watch.

Calculate the frequency of the organ-pipe from the second formula given above, adding or subtracting the beats according as the siren was brought up or down to beat with the organ-pipe.

Take several observations, and average the calculated results.

Check the result by comparing the pipe with a standard tuning-fork of nearly the same period.

This may be done by the method of beats, the fork being loaded to find which note is higher.

Precautions.—Do not work the bellows with jerks or they may burst.

Be careful that the gasometer is never hard up against the stop at the top of its gauge or the water will be forced out of the gasometer and gauge.

See that the siren is well oiled, and the pivot bearings properly adjusted.

Note.—In this experiment the greatest difficulty was

encountered in keeping the speed of the siren sufficiently constant during observation. For this a speed-governor, similar to those used in electric power-supply meters,—notably those of Perry & Elihu Thompson,—was employed.

It may seem that the list of apparatus given above is rather an elaborate one for the performance of this experiment, but it was found impossible to obtain good results with the apparatus usually employed.

Example.—Enter results thus:

Siren Revolutions.	Time.	Beats.	p.	Frequency.
1500 1000 1000	43.4″ 30.0″ 29.3″	− 217 + 436 − 219	15	513 510 507
Mean value of *F*........................				510

Blank to be filled in by student.

Siren Revolutions.	Time.	Beats.	p.	Frequency.
Mean value of *F*........................				

4. TO COMPARE THE FREQUENCY OF TWO NEARLY IDENTICAL FORKS BY BEATS.

References.—Barker, p. 252; Watson, p. 426; Anthony and Brackett, p. 166; Carhart, pt. I. p. 159 (4); Knott, p. 286; Nichols and Franklin, pp. 150, 175; Ames, pp. 163, 189; Hastings and Beach, p. 552.

Apparatus Required.—Two forks of nearly the same pitch, mounted on suitable resonators, or two identical forks and a pair of weights for loading one of them. The weights should be provided with reference-points for the determination of their positions on the prongs of the forks. In addition a rubber hammer, a stop-watch, a centimeter scale.

Theory of Experiment.—If two notes of nearly identical pitch be sounded together, a peculiar fluctuating effect is produced. Alternate intervals of comparative silence and bursts of sound are heard. These bursts of sound are called beats, and the notes are said to beat together.

Let the frequency of the forks be n_1 and n_2 per second.

Then $$n_1 = n_2 + p, \quad \ldots \ldots \quad (1)$$

where p is the number of beats,

or $$n_1 - n_2 = \frac{N}{t}, \quad \ldots \ldots \quad (2)$$

where N is the number of beats heard in t seconds. For while the first fork makes n_1 vibrations the other makes p more, and therefore in $1/p$ of a second the second fork executes one whole vibration more than the other. At the

end of that time, therefore, the sound will be reinforced, as well as at the end of every succeeding $1/p$ part of a second. Midway between these points the second fork has just gained a half vibration on the other, the two forks are in opposition, and there will therefore be an interval of silence.

It follows that if the number of beats or loud points be counted in a given time, the difference between the frequencies is completely determined.

Practical Directions.—It is more convenient to have one of the forks driven electromagnetically. If such a fork is available, it can very well be used as the standard. Load the other fork near the ends of the prongs by means of the small weights provided, until the beats are such as can be easily counted.

Count the time of, say, 20 beats, if the loaded fork vibrates long enough. Measure the distance of the weights from the ends of the prongs and calculate the difference between the forks by formula (2).

There will be overtones immediately after the excitement of the loaded fork; these, however, soon die away, and at any rate do not much interfere with the perception of the beats.

To determine whether the loaded fork is flatter or sharper than the standard, raise the loads a very little.

Redetermine the beats per second, and if there are fewer to the second, the loaded fork was higher; if more, *vice versa*.

If the forks are not identical, load the higher one until no beats are heard.

Measure the distance of the loads from the ends of the prongs.

The gradual dying away of the note must not be confounded with the very slow beats which occur as the forks approach unison.

Example.—Enter results thus:

Frequency of Standard.	Distance of Weight from End.	Time.	Beats.	Frequency of Loaded Fork.
512	6 cm.	20″	50	514.5
	5.5 "	22″	45	514.0
	4.5 "	25″	10	512.4

Blank to be filled in by student.

Frequency of Standard.	Distance of Weight from End.	Time.	Beats.	Frequency of Loaded Fork.

5. OPTICAL TUNING.—TO COMPARE THE FREQUENCIES OF TWO NEARLY IDENTICAL TUNING-FORKS BY LISSAJOUS FIGURES.

References.—Watson, p. 400; Barker, p. 262; Hastings and Beach, pp. 529 and 571; Nichols and Franklin, vol. III. p. 152; Anthony and Brackett, p. 180.

Apparatus Required.—A pair of identical tuning-forks (with a frequency of about 100 D. V.) provided with mirrors and supports; a dark lamp with a pin-hole in the chimney; a telescope; a rubber exciting-hammer; a stop-watch.

Theory of Experiment.—The theory is substantially identical with that of the preceding experiment (Comparison of Forks by Beats), the only difference being in the method of observation. The beats in this case are detected with the eye instead of the ear.

Practical Directions.—(1) *Composition along the Same Straight Line.*—Clamp the forks to their supports so that they may vibrate in the same plane.

Bring up the lamp to about a meter from one of the mirrors, and adjust its position so that an image of the illuminated pin-hole is seen by the eye when placed level with the pin-hole and about one-third of a meter at one side of it.

Place the other mirror to face the former at about one meter distance, and so that the image of the pin-hole as seen in the former is intercepted by the latter.

Place the telescope at about four meters from the second mirror, and focus it on the image seen in this mirror. The image as seen in the telescope may be a trifle blurred owing to defects in the mirrors.

If possible, clamp the supports of the forks firmly to the table, or if not, the supports must be held tightly by the hand when exciting the forks, to prevent them moving.

Excite each of the forks by a blow from the hammer.

If the forks are in unison, a luminous straight line of very gradually diminishing length will be seen in the telescope. This image reduces eventually, on the cessation of both forks, to the image of the pin-hole, without having undergone any elongation whatever from the beginning.

Now load one of the forks and re-excite them both.

A straight line of periodically varying length will be seen. If the vibrations of the forks have equal amplitudes, the length will vary from a mere point, the original image of the pin-hole, to a line whose length is equal to the magnified sum of the amplitudes.

The point corresponds to the interval of silence in audible beats, and the long line to the burst of sound.

If the amplitudes are not the same, a line, equal to the

magnified difference between the amplitudes, will be observed to be the minimum length of the fluctuating line. This accounts for the period of only comparative silence observed in audible beats.

Count fifty of these variations from point to point or minimum length to minimum length, and calculate by the formula

$$n_1 - n_2 = \frac{N}{t},$$

the difference between the forks, where n_1 and n_2 are the frequencies of the forks, and N is the number of beats in the time t.

The time between any two consecutive minimum lengths corresponds to the time of one audible beat.

Measure the distance of the load index from the end of the fork-prong. Repeat the observation several times, changing the position of the load in each case.

(2) *Composition at Right Angles.*—Without altering the load, turn the forks in their supports so that they may vibrate at right angles. This is most easily done by turning them to vibrate at 45° to the table, the axes of the mirrors being in the same horizontal plane.

Adjust, as before, until the image from the second mirror is seen in the telescope. If the forks have only slightly different periods, a figure will be seen, changing from a straight line at 45° to the horizontal, through an ellipse, to a straight line at right angles to the former line, and then through another ellipse back to the original line, see Fig. 2.

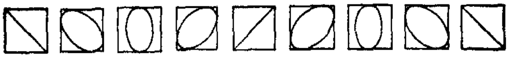

FIG. 2.

The time taken to make a complete cycle is the time of one beat.

As in (1), count fifty of these complete cycles. They should be found to correspond with the periodic changes in the previous comparison.

Repeat the observations with loads in same positions as in (1).

(3) Procure two forks which are not identical, and load the higher one to unison with the other by observing when there is no periodic change in the figure.

Record the position of the load.

Precautions.—Do not touch the mirrors.

On no account is the fork to be excited by striking the prong carrying the mirror.

Example —*Enter results thus:*

Distance of Load from End of Fork.	Periodic Changes.	Time.	$n_2 - n_1$
(1) 3.5 cm.	50	80.0″	.6
(2) 4 cm.	50	60.2″	.8
(3) 5 cm.	50	40.4″	1.25
(4) 6 cm.	50	20.0″	2.5
(5) 7 cm.	50	10.4″	4.8

Blank to be filled in by student.

Distance of Load from End of Fork.	Periodic Changes.	Time.	$n_2 - n_1$

6. TO DETERMINE THE VELOCITY OF WAVES IN A STRETCHED STRING.

References.—Watson, p. 358; Knott, p. 261; Hastings and Beach, p. 523; Nichols and Franklin, vol. III. p. 161; Ames, p. 171.

Apparatus Required.—A long steel wire; a stop-watch; a pulley; weights for varying the tension; a tape-measure.

Theory of Experiment.—When a stretched string is made to vibrate, the velocity of the wave motion along the string is given by the equation

$$v = \sqrt{\frac{T}{m}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where T is the tension, and m the mass per unit length.

If, therefore, the tension and unit mass be known, the velocity can be calculated.

If the time of transmission of the wave from one end of the string to the other be observed, the velocity calculated from (1) can be verified.

Thus if l be the length of the string, t the time of transmission of a wave from one end of the string and back, then

$$v = \frac{2l}{t}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Practical Directions.—Weigh a known length of the wire, and find m.

Fasten one end of the long wire to the wall of the room, passing the other end over a pulley fixed some distance away.

To the end which passes over the pulley fasten an attachment for holding weights, and put on a weight, W, of about one kilogram.

Strike the wire lightly near the end. A wave motion will be now transmitted along the wire and back.

Place the finger on the wire about one inch from the pulley. The return of the wave can be distinctly felt by the finger. The return of the wave can also be observed by the eye.

Start the stop-watch just as the first pulsation is felt, and take the time of fifty returns.

Measure the length of the wire in centimeters.

If the length be measured in feet, reduce to inches and multiply by 2.54 for centimeters.

Express the tension, T, in dynes.

If the weight be in pounds, multiply by 456.3 × 981 for dynes.

Calculate v from the formula

$$v = \sqrt{\frac{T}{m}}.$$

Calculate v from the observation of fifty returns.

Repeat the observation three times with different weights.

Example.—Enter the results thus:

Obs.	m	l	Time of Fifty Returns.	T	v from Observations.	v from $v = \sqrt{\frac{T}{m}}$.
1st obs.	.02275	1938	20.5″	2032622	9452	9452
2d obs.	.02275	1938	27.2″	1136471	7171	7089

Blank to be filled in by student.

Obs.	m	l	Time of Fifty Returns.	T	v from Observations.	v from $v = \sqrt{\frac{T}{m}}$.

7. TO DETERMINE THE PITCH OF A FORK BY THE TRACE OF ITS VIBRATION ON A SMOKED FALLING PLATE.

References.—Barnes's Practical Acoustics, p. 75.

Apparatus Required.—A tuning-fork of fairly high frequency rigidly fixed to a suitable support; a suitable glass plate; a pair of dividers; a centimeter scale; a hog's bristle; a plucker for vibrating the fork.

Theory of Experiment.—If a smoked glass-plate be let fall freely, so as to receive the trace of a vibrating tuning-fork, the trace on the plate will be a sinuous line. The vibrations will be very close at the bottom of the plate, lengthening out toward the top, provided the plate at starting is in contact with the tracer on the fork, due to the slow motion of the plate at first.

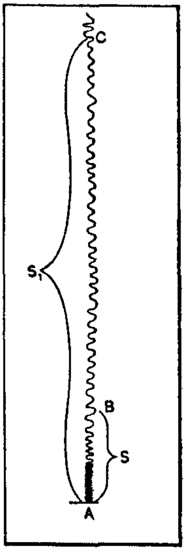

FIG. 3.

Suppose at starting the tracer of the fork be at A, and that the vibrations can be counted between the points B and C. Denote the length AB by S, and AC by S_1.

If t be the time it would take the point B to fall to the tracer, then

$$S = \tfrac{1}{2}gt^2 \qquad (1)$$

Similarly,

$$S_1 = \tfrac{1}{2}gt_1^2, \qquad (2)$$

t_1 being the time it takes the point C to reach the tracer.

Hence $$t = \sqrt{\frac{2S}{g}},$$

and $$t_1 = \sqrt{\frac{2S_1}{g}},$$

and therefore $$t_1 - t = \sqrt{\frac{2S_1}{g}} - \sqrt{\frac{2S}{g}}, \quad . \quad . \quad . \quad . \quad (3)$$

where $t_1 - t$ is the time it takes the tracer to pass from B to C.

If v be the number of vibrations between B and C, then the vibration frequency of the fork is given by the equation

$$n = \frac{v}{t_1 - t}. \quad . \quad . \quad . \quad (5)$$

Since v can be counted, S and S_1 measured, g is known, n can be calculated.

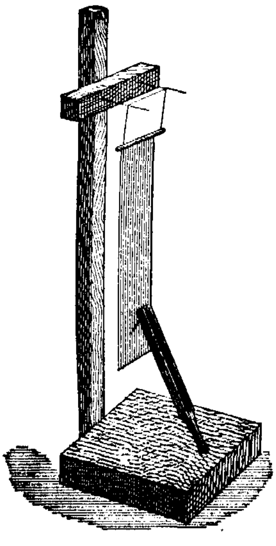

FIG. 4.

Practical Directions.—Clamp the fork-stand to the table. Fasten a bristle to the fork at an angle downward of about 45°.

Smoke the glass plate by passing it rapidly back and forth through a smoky paraffine-lamp flame. The glass plate should be quite thick, so as to make it strong and comparatively heavy.

Hang the plate, by means of a loop of cotton thread, to the supports provided for the purpose.

Adjust it carefully so that it just touches the bristle when hanging vertically with its smoked surface in the plane of

vibration of the fork. Fig. 4 shows apparatus when complete.

Set the fork vibrating by gently pulling off the plucker. If the plate dance about, its plane is not parallel to the plane of vibration, and must be adjusted.

When properly adjusted, sever the suspension by touching it midway between the supports with a lighted taper.

A clear continuous trace will now be on the plate if the adjustments have been made with sufficient care.

Select two points corresponding to B and C, between which the vibrations can be counted.

Measure the distances S and S_1 between the first point of contact and the points B and C.

Count the vibrations between B and C.

Calculate n from the observations.

Repeat the observations three times.

Example.—Enter results thus:

$g = 981$.

Observations.	S	S_1	v	n
1st	1	10	106	1075
2d	1	9 5	102	1080
3d	1	10.3	108	1074
Mean value of n..................................				1076

Blank to be filled in by student.

Observation.	S	S_1	v	n
1st				
2d				
3d				
Mean value of n..................................				

8. LAWS OF VIBRATING STRINGS.—MELDE'S METHOD.

References.—Knott, p. 275; Watson, p. 396; Ames, pp. 173–175; Nichols and Franklin, p. 160; Hastings and Beach, p. 563; Carhart, pt. I. p. 188.

Apparatus Required.—An electrically driven tuning-fork with cord attachment; a small pulley fixed to an upright stand so that a cord can be stretched over it; some small weights; a piece of linen thread or small silk cord.

Theory of Experiment.—If a string of length l be made to vibrate under a tension T, we have seen that the laws of vibration are expressed by the equation

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where n is the number of vibrations per second, l the half wave-length of the vibration in the string, T the tension, and m the mass per unit length.

If a cord attached to the prong of a tuning-fork be stretched under a tension and over a pulley, and the fork be set vibrating, the cord between the pulley and the fork will be thrown into vibrating segments, as shown in Fig. 5, when its length is properly adjusted.

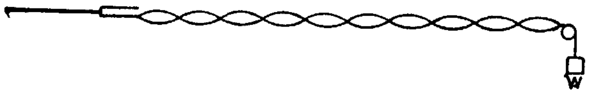

FIG. 5.

(1) The length of the string between the two successive nodes gives the value l; therefore n, the vibrating frequency of the fork, can be calculated, since T and m are known quantities.

(2) Since $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$ for a tension T,

and also $n = \frac{1}{2l_1}\sqrt{\frac{T_1}{m}}$ for a tension T_1,

then $$\frac{\sqrt{T}}{2l} = \frac{\sqrt{T_1}}{2l_1},$$

or $$\frac{T}{l^2} = \frac{T_1}{l_1^2}. \qquad (2)$$

By varying the weights, therefore, the law $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$ can be experimentally verified.

Practical Directions.—Weigh a known length of the string and thus find m, the mass per centimeter.

Attach the cord to the prong of the fork, and stretch it over the pulley.

Attach 30 or 40 grams weight to the end of the cord.

Set the fork vibrating.

Measure the length of the string between several nodes, and obtain the average length, l.

Observe the weight on the string, and reduce to dynes.

Calculate n from the formula

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}.$$

Now vary the weights three different times, and repeat the observations for l.

Then $$\frac{T}{l^2} = \frac{T_1}{l_1^2} = \frac{T_2}{l_2^2}.$$

Example.—Enter results thus:

Obs.	T	l	n	$\frac{T}{l^2}$
1st.........	127530	12.90	512	766
2d.........	156960	14.30		767
3d.........	88290	10.75		763

Blank to be filled in by student.

Obs.	T	l	n	$\frac{T}{l^2}$

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9. TO DETERMINE THE VELOCITY OF SOUND IN VARIOUS MEDIA BY MEANS OF KUNDT'S TUBE.

References.—Watson, p. 423; Barker, p. 223; Nichols and Franklin, vol. III. p. 160; Carhart, pt. I. p. 210; Ames, p. 185; Hastings and Beach, p. 561; Anthony and Brackett, p. 162.

Apparatus Required.—Kundt's tube with some fine light powder, such as cork-filings; a wet silk cloth; a centimeter scale; a thermometer; a drying-tube; a meter or so of gas-tubing; a pair of bellows.

Theory of Experiment.—If the apparatus, arranged as a Kundt's tube, be supported horizontally and some light powder evenly distributed over the bottom of the tube, the powder will arrange itself into heaps when the rod is rubbed sufficiently to emit a note.

The rubbing produces longitudinal vibrations in the rod, which are communicated to the air in the tube as compressions and rarefactions. The powder is forced away from the places of motion, the loops, to the points of rest, the nodes.

If the rod be rigidly fixed at its centre by a clamp, its ends will be at the middle of consecutive loops, the clamp being at the intervening node.

The length of the rod is therefore equal to one-half of the wave-length of the note emitted. Denote the length of the rod by l. The distance from heap to heap, d, is equal to one-half of the wave-length of the same note in the gas.

These lengths are described in equal times, since the gas in the tube vibrates in unison with the rod.

The velocity of sound in any medium is equal to the wave-length multiplied by the number of vibrations per second. Therefore

$$\frac{v_1}{v_2} = \frac{l}{d}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

v_1 and v_2 being the velocities in the rod and gas, respectively.

Knowing the temperature, t_0, of the air in the tube, the velocity in it may be obtained from the formula

$$v_2 = 33250 \sqrt{1 + .003665t}, \quad . \quad . \quad . \quad . \quad (2)$$

33250 being the velocity at 0° C., and therefore v_1 can be calculated.

Practical Directions.—See that the tube is clean and dry.

Clamp the rod in the middle.

Pull out the adjustable plunger, and shake into the tube either dry cork-filings or amorphous silica.

Replace the plunger, and support the whole horizontally.

The powder should lie in a thin coating along the bottom of the free part of the tube.

Open both stop-cocks and connect one of them to the bellows through a drying-tube.

Force dry air in for some time before closing the cocks.

The rod, if glass, may be excited by stroking its free half with wet silk; but in the case of brass or other metals resined chamois will be found better.

If after the first rubbing the nodes in the tube are not well defined, adjust the length of the column of air by the plunger and repeat the rubbing. Continue the adjustment until the nodes are sharply defined.

When the nodes have become sharp, measure the distance to each from one end of the tube. Subtract the distance of the middle one from the first, the distance of the next one beyond the middle from the second, and so on to the last one. Take the mean of these results and divide by the number of loops contained. This should give a good mean value for a half wave-length of the vibrating air in the tube.

Calculate the velocity in the material of the rod by formula (1), having substituted the velocity in air corrected by formula (2) for the temperature of the room.

The velocity in dry air at 0° C. may be taken as 33250 cm. per second.

The temperature of the air may be obtained with sufficient accuracy from a thermometer on the table near the tube.

Take observations for both the glass and brass rods.

Example.—Enter results thus:

Temperature of air .... 16.4° C.

Hence $v_t = 33250 \sqrt{1.060024.}$
$= 34230$ cm. per sec.

No. of Node.	Brass Rod, l = 108 cm. Distance from Piston.	Length of Four Loops.	Between Nos.	Glass Rod, l = 108 cm. Distance from Piston.	Length of Five Loops.	Between Nos.
1	10.5	....	....	7.5	....	....
2	21.0	....	....	15.0	....	....
3	32.0	....	....	22.5	....	....
4	42.5	....	....	30.0	....	....
5	52.6	42.1	5 and 1	37.5	....	....
6	63.0	42.0	6 " 2	45.0	37.5	1 and 6
7	73.0	41.0	7 " 3	53.0	38.0	2 " 7
8	83.0	40.5	8 " 4	60.5	38.0	3 " 8
9	....	....	....	68.0	38.0	9 " 4
10	....	....	....	75.5	38.0	10 " 5
•	Mean length of 1 loop =	41.4 / 10.35	....	....	37.7 / 7.59	....

$$v_1 \text{ (for glass)} = \frac{108}{7.59} \times 34230 = 487800 \text{ cm. per sec.}$$

$$v_1 \text{ (for brass)} = \frac{108}{1035} \times \text{"} = 356200 \text{ " "}$$

Blank to be filled in by student.

Temperature of air ....

No. of Node.	Distance from Piston.	Length of Loops.	Between Nos.	Distance from Piston.	Length of Loops.	Between Nos.
	Mean length of 1 loop =					

10. TO DETERMINE THE VIBRATION FREQUENCY OF A TUNING-FORK BY MEANS OF A PENDULUM-CHRONOGRAPH.

Apparatus Required.—A suitable pendulum; a tuning-fork; a stop-watch; a centimeter scale; a set square; a rubber hammer; a suitable clamp and stand for mounting the fork.

Theory of Experiment.—If a pendulum be made to swing past a fork vibrating vertically, the pendulum and the fork being so arranged that the vibration of the fork can be traced by means of an attached bristle on a smoked glass surface attached to the pendulum, then the path of the point attached to the vibrating fork will be a sinuous line as ABC, A and C marking the beginning and end of the swing respectively. See Fig. 6.

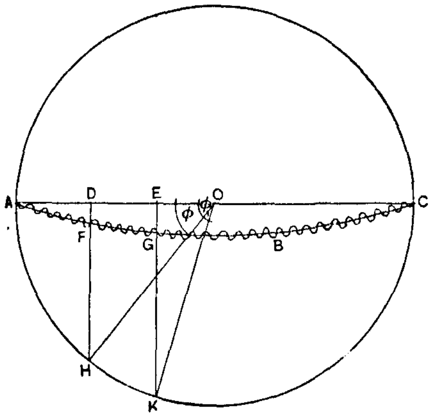

FIG. 6.

If the arc through which the pendulum swings be short, its motion may be regarded as a simple harmonic motion along the line AC.

If the time of motion from F to G, that is, from D to E, can be found, and the number of vibrations of the fork between these two points counted, then the number of vibrations per second can be calculated.

Let $AHKC$ represent the auxiliary circle. Take D and E, two points on the line AC, on the same side of the centre. Erect perpendiculars DFH and EGK. Join H and K to the centre of the circle, O.

Denote the angle HOA by ϕ, AOK by ϕ_1, the periodic time of the pendulum by P, AD by y, AE by y_1, AC by $2a$, the time of motion from A to D (that is, from A to F on the arc, or from A to H on the circle) by t, the time of motion from A to E by t_1, the vibration frequency of the fork by n, and the number of vibrations between F and G by v.

Since P is the periodic time of the pendulum,

$$\phi = \frac{(360 \, . \, t)}{P},$$

$$\phi_1 = \frac{(360 \, . \, t_1)}{P}.$$

Hence $$t_1 - t = \frac{P(\phi_1 - \phi)}{360}, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$t_1 - t$ being the time from F to G.

Now, $$n = \frac{v}{t_1 - t} = \frac{v \, . \, 360}{(\phi_1 - \phi)P}. \quad . \quad . \quad . \quad (2)$$

Also, $$\cos \phi = \frac{OD}{OH} = \frac{(a - y)}{a},$$

and $$\cos \phi_1 = \frac{OE}{OK} = \frac{(a - y_1)}{a}.$$

If a, y, y_1 be measured, ϕ and ϕ_1 can be found from the mathematical tables. Hence, n can be calculated from equation (2).

Practical Directions.—Smoke the surface of the glass plate by passing it rapidly back and forth through the flame of an oil-lamp.

Replace the plate in the clamp provided on the pendulum for the purpose.

Adjust the stop-catches on either side of the pendulum, so that when it is released from the one it swings across and just catches on the other.

Clamp the fork whose rate is to be determined, so that it vibrates in a vertical plane.

Adjust the fork and plate so that the bristle of the former just touches the latter throughout its entire swing. When the pendulum is held in one catch, the bristle should touch the glass plate near one end.

Release the pendulum from the stop so that it swings past the fork. The bristle will describe the arc of a circle.

Bring the pendulum back to its original position and excite the fork by a blow from a rubber hammer (if it be not driven magnetically.)

Release the pendulum from the stop again, and over the arc already described a sinuous curve will be traced.

To find P.—Having obtained a record of the vibrations of the fork, without altering the adjustments of the pendulum set it swinging, and take carefully to the fifth of a second the time of 100 swings; that is, 50 complete oscillations. From this observation calculate P.

To find ϕ and ϕ_1.—Take out the plate of glass. Carefully join the extreme points of the arc by a straight line. At two points (as D and E in Fig. 6) in this line, by means of a set square, erect perpendiculars.

Measure carefully, by means of a pair of dividers and

millimeter scale, the lengths corresponding to y, y_1, and $2a$. Since the values of ϕ and ϕ_1 depend only on y, y_1, and a, they can at once be calculated.

To find v.—Count carefully to the tenth of a vibration the number of vibrations between the two perpendiculars.

Precautions.—(1) Be careful not to break the glass plate when smoking it; it must be moved rapidly back and forth to prevent uneven heating.

(2) Be careful to adjust the stops of the pendulum so that the whole arc will be on the plate, otherwise the length, a, will not represent the amplitude of vibration.

Example.—Enter results thus:

Time of 50 oscillations, 97.00″

Therefore	P	=	1.94	
	y	=	7.3	
	y_1	=	14.0	
	$2a$	=	32.4	
	v	=	25.4	
	$\cos\phi$	=	$\frac{8.9}{16.2}$	$\therefore \phi = 56° 6'$
	$\cos\phi_1$	=	$\frac{2.2}{16.2}$	$\therefore \phi_1 = 82° 48'$
Hence,	n	=	128.7.	

Blank to be filled in by student.

Time of 50 oscillations,

P	=		
y	=		
y_1	=		
$2a$	=		
v	=		
$\cos\phi$	=		ϕ =
$\cos\phi_1$	=		ϕ_1 =
n	=		

LIGHT.

11. TO COMPARE THE INTENSITIES OF TWO SOURCES OF LIGHT. BUNSEN'S PHOTOMETER.

References.—Ames, p. 391; Knott, p. 244; Hastings and Beach, p. 717; Watson, p. 500; Nichols and Franklin, vol. III. p. 117; Anthony and Brackett, p. 447; Barker, p. 383.

Apparatus Required.—A photometer; an incandescent lamp; a standard candle.

Theory of Experiment.—"When a grease-spot is made on a piece of bibulous paper, the spot appears translucent. If the paper be illuminated by a light in front, the spot appears darker than the surrounding space; if, on the other hand, it be illuminated from behind, the spot appears light on a dark ground." If, therefore, the two sources of light to be compared be placed one on each side of the paper, the line joining them being at right angles to the plane of the paper, and the distances adjusted until as seen from both sides the greased spot and the rest of the paper appear to have the same brightness, the intensity of illumination of the two sources of light in the direction of the paper is the same, and therefore

$$\frac{I}{I_1} = \frac{r^2}{r_1^2},$$

r and r_1 being respectively the distances from the paper of the lights whose intensities are I and I_1.

Practical Directions.—Fix the lamp and candle on the photometer at a distance of about 50 cm. apart. Adjust the grease-spot until the point of equal illumination is reached.

A substitute for the grease-spot may be made as follows:

Take two rectangular blocks of paraffine wax of equal dimensions. Between two corresponding faces place a strip of tin-foil, adjusting so that two other faces are in the same plane. Place the combination between the sources of light so that the lights fall perpendicular to the surfaces parallel to the foil. Adjust until the two faces in the same plane are the same shade.

Record the observations of distances r and r'.

Repeat the observations for three positions of the lamp, filament side *on*, filament edge *on*, filament end *on*.

Repeat the observations again with the lamp and candle 75 cm. apart, and again 100 cm. apart.

Example.—Enter results thus:

Position of Lamp.	Distance between Lamp and Candle.	r. (Lamp.)	r_1. (Candle.)	$I/I_1 = r^2/r_1^2$.
Filament side on	50	39.7	10.3	14.9 (1)
" edge "		38.0	12.	10.1 (2)
" end "		34.9	15.1	5.3 (3)
" side "	75	59.5	15.5	14.7 (1)
" edge "		57.0	18.0	9.9 (2)
" end "		52.2	22.8	5.2 (3)
" side "	100	79.2	20.8	14.5 (1)
" edge "		76.1	23.9	10.2 (2)
" end "		69.9	30 1	5.5 (3)

Blank to be filled in by student.

Position of Lamp.	Distance between Lamp and Candle.	r. (Lamp.)	r_1. (Candle.)	$I/I_1 = r^2/r_1^2$.

12. TO COMPARE THE INTENSITIES OF TWO SOURCES OF LIGHT. RUMFORD'S PHOTOMETER.

References.—The same references as in previous case.

Apparatus Required.—A Rumford photometer; a wax candle; an electric lamp.

Theory of Experiment.—The intensity of illumination on a given surface produced by a source of light is inversely as the square of the distance from the source of light, and directly as the cosine of the angle which the luminous rays make with the normal to the illuminated surface.

If the two sources of light be placed in front of an upright rod behind which is a screen, each will project on the screen a shadow of the rod.

By altering the relative position of the two sources of light the intensities of the two shadows may be made the same. Then, since the shadow of each is illuminated by the other, the illumination of the screen due to each light is the same. Suppose I to be the intensity of the one source of light, r its distance from the screen, α the angle which the direction of the beam makes with the normal to the screen; then the illumination due to this source is equal to

$$\frac{I \cos \alpha}{r^2}.$$

If I_1, α_1, r_1 be corresponding values for the second source, then this illumination is equal to

$$\frac{I_1 \cos \alpha_1}{r_1^2};$$

and since these are equal, we have

$$\frac{I \cos \alpha}{r^2} = \frac{I_1 \cos \alpha}{r_1^{\,2}}, \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

or

$$\frac{I}{I_1} = \frac{r_2 \cos \alpha_1}{r_1^{\,2} \cos \alpha}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

If $\alpha = \alpha_1$, that is, if the directions of the luminous rays be equally inclined to the surface,

$$\frac{I}{I_1} = \frac{r_2}{r_1^{\,2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

Practical Directions.—Attach the candle and lamp to the sliding attachments provided for the purpose.

Adjust the angle between the sources of light until the edges of the shadows just meet.

Adjust the screen so that the normal to its surface, through the rod, bisects the angle between the directions of the two luminous beams.

Adjust the distances from the screen until the shadows are equally illuminated.

This last may be rather difficult to determine owing to the fact that the two sources, being different, emit differently colored rays in different proportions, so that the colors of the shadows are not quite the same. In that case the student must use his judgment as to the equal densities of the shadows.

Measure the distances of the candle and the lamp from the screen, r_1 and r_2.

Make the comparison with the filament of the lamp first parallel to the screen, then at right angles, and finally with the lamp end on. In all three cases repeat the observations for three different positions of the candle and lamp.

Example.—Enter results thus:

Position of Lamp.	r	r_1	$r^2/r_1^2 = I/I_1$.
Filament side on	39.7	10.3	14.9
" edge "	38.0	12.0	10.1
" end "	34.9	15.1	5.3
" side "	59.5	15.5	14.1
" edge "	57.0	18.0	9.9
" end "	52.2	22.8	5.2
" side "	79.2	20.8	14.5
" edge "	76.1	23.9	10.2
" end "	79.9	30.1	5.5

Blank to be filled in by student.

Position of Lamp.	r	r_1	$r^2/r_1^2 = I/I_1$.

13. TO VERIFY THE LAW OF REFLECTION.

References.—Knott, p. 252; Nichols and Franklin, vol. III. p. 225; Carhart, pt. I. p. 241; Jones, p. 137; Hastings and Beach, p. 611; Ames, p. 405; Anthony and Brackett, p. 405; Watson, p. 446; Barker, p. 406.

Apparatus Required.—A drawing-board; a piece of silvered glass about 10 cm. long and 1 cm. wide; a clip for holding the glass in a vertical plane; half a dozen pins.

Theory of Experiment.—If a plane mirror be held in a vertical plane and a luminous point placed in front of it, an image will be seen formed behind the mirror, no matter from what point of the mirror it may be reflected.

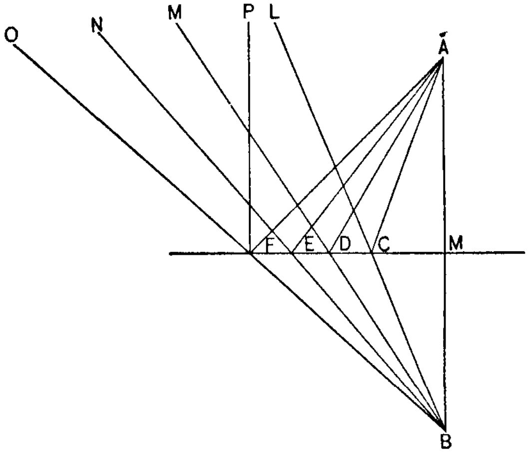

FIG. 7.

Let A be a luminous point.

Suppose it to be reflected successively from the points C, D, E, F, on the mirror.

If the lines LC, MD, NE, OF, be drawn marking the directions in which the image is seen from the successive points, it will be found that they all pass through a point behind the mirror, the point where the image is seen.

Denote this point by B.

If AB be joined, then by measurement it will be found that AM is equal to MB, and that AB is perpendicular to FM.

From this it follows, obviously, that the angle of incidence is equal to the angle of reflection; e.g., the angle PFA is equal to the angle PFO, where PF is perpendicular to FM.

Practical Directions.—(1) Fasten a sheet of ruled foolscap paper upon a drawing-board. Stick a number of pins vertically into the board along one of the lines of the paper.

Let these be represented by the points *F*, *E*, *D*, *C*, in the preceding diagram.

Place the mirror in the same vertical plane, with its silvered face touching the pins, and adjust it so that its edges are parallel with the paper, the lower edge being about one centimeter above it.

Stick another pin, also vertically, at a point corresponding to *A* in the diagram.

Reflect this pin successively from the points *C*, *D*, *E*, *F*. This can be done by getting the image of the pin at *A* in a line with the pin at each of the points, and marking the direction in each case by means of pins at *L*, *M*, *N*, *O*.

Now remove the mirror, and join *LC*, *MD*, *NE*, and *OF*, producing them to meet. The same pin will do for marking the points *L*, *M*, *N*, *O*. If the observations are carefully taken, the lines *LC*, *MD*, *NE*, and *OF* will meet in a point. To show the results, draw the complete diagram.

(2) It follows geometrically that if the angle of incidence be equal to the angle of reflection, the position of the image is behind the mirror at a distance equal to the distance the object is in front, and that the line joining the object and image is perpendicular to the mirror.

Hence the law of reflection can be verified experimentally by measuring these distances.

Place the mirror as before in a vertical plane so that the lower edge is about a centimeter above the paper.

Stick a pin vertically at a distance of 10 to 15 cm. in front of the mirror.

Now while observing the image of the pin behind the mirror, stick another pin so as to coincide with this image.

In order to determine whether the two really coincide, move the eye at right angles, back and forth, to the direction of the two pins. If the second pin coincides with the image, the two will appear to move together, otherwise they will appear to move away from each other.

Having adjusted the pin to a proper position, measure the distances of the two pins from the silvered side of the mirror, and verify with a set-square the perpendicularity in each case.

The small differences may be due to the unevenness of the glass, or to faulty observations.

Take at least six readings.

Example.—Enter results thus:

(1) Show complete diagram.

(2) Show diagram and table.

Dist. of Pin from Mirror.	Dist. of Image behind.	Difference.
10 cm.	10.2	.2
12.1 "	12.2	.1
15 "	15.2	.2
17.2 "	17	.2

Blank to be filled in by student.

Dist. of Pin from Mirror.	Dist. of Image behind.	Difference.

14. TO FIND THE ANGLE OF A PRISM. PIN METHOD.

References.—Watson, p. 458; Barker, p. 430; Knott, p. 263; Nichols and Franklin, vol. III. p. 38; Ames, p. 429.

Apparatus Required.—A prism; a pair of dividers; a centimeter scale; a book of logarithmic tables.

Theory of Experiment.—Suppose from a luminous point A a ray of light fall upon the edge of a prism POQ, and

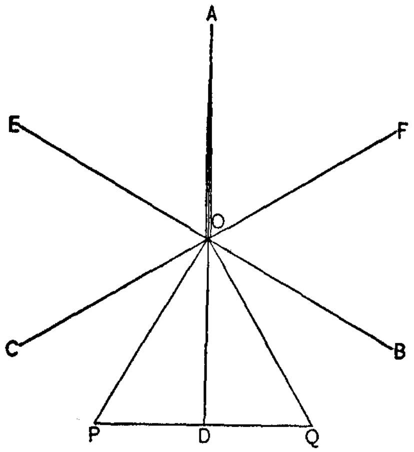

FIG. 8.

that it is reflected from the side OQ at the edge O along the line OB, and from the side RO along the line CO. Then, since the angle of incidence is equal to the angle of reflection,

$$AOE = EOC,$$

and

$$AOF = FOB,$$

EO being perpendicular to PO, and FO to QO.

If AO be produced to D, it is evident geometrically that

the angle COP is equal to the angle POD, and that the angle BOQ is equal to the angle DOQ.

Therefore COB is double POQ, that is, double the angle of the prism. Hence if COB be measured, the angle of the prism is found.

Practical Directions.—Describe a circle with a radius of 10 cm., centre C. Place the prism with the angle to be found at the centre of the circle. Stick a pin vertically at a

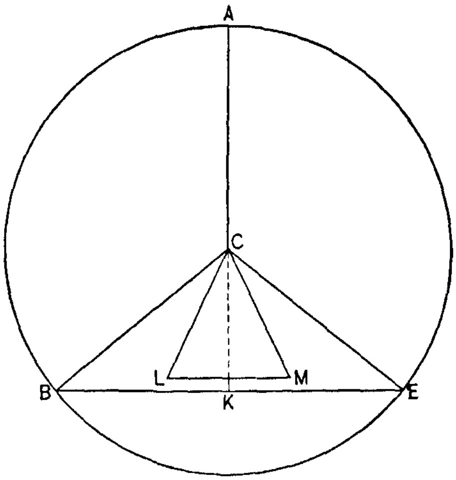

FIG. 9.

point A on the circumference, so that the line AC approximately bisects the angle of the prism.

Mark the direction of the reflection of the pin A by sticking a pin at E, so that E, the image of A, and the edge C are in a straight line.

Mark the point B in the same way.

Now BCE is equal to twice LCM.

If BE be bisected at K and joined to C, then CK bisects BCE, and also is perpendicular to BE.

Hence the angle BCK is equal to the angle LCM.

Now, $$\sin BCK = \frac{BK}{BC} = \frac{BE}{2BC}.$$

Hence measure BE, divide by twice BC, and the result is the sine of the angle LCM.

By reference to the table of natural sines the angle may be found.

Measure the three angles of the prism.

Example.—Enter results thus:

RADIUS OF CIRCLE 10 CM.

Angle.	BE	Sin I.	I.
1st angle	17.33	.8665	60° 3′
2d angle	17.40	.870	60 30
3d angle	17.20	.860	59 24
Sum of angles of prism			179° 57′
Difference from 180°			− 3′

Blank to be filled in by student.

Angle.	BE	Sin I.	I.
1st			
2d			
3d			
Sum of angles of prism			
Difference from 180°			

15. TO FIND THE REFRACTIVE INDEX OF GLASS.

References.—Barker, p. 424; Watson, p. 468; Carhart, p. 257; Ames, p. 423; Nichols & Franklin, vol. III. p. 36; Knott, p. 259; Hastings and Beach, p. 615; Anthony and Brackett, p. 407.

Apparatus Required.—A glass plate; a drawing-board; a sheet of white paper; a few pins; a centimeter scale; a set-square.

Theory of Experiment.—If a ray of light *FF* fall upon a plate of glass *ABCD*, then on passing into the glass it is

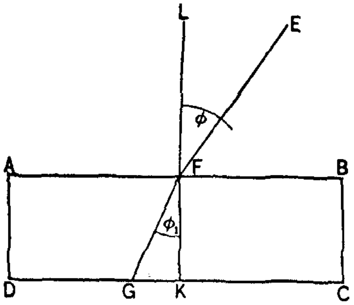

FIG. 10.

bent along a line *FG*, making the angle *GFK* with the normal *LF*, less than the angle *LFE*. The ratio

$$\frac{\sin LFE}{\sin GFK}$$

is called the refractive index of the glass.

Denoting the refractive index of glass by μ, the angle *LFE* by ϕ, and the angle *GFK* by ϕ_1,

$$\mu = \frac{\sin \phi}{\sin \phi_1}.$$

If ϕ and ϕ_1 be determined, μ can be calculated.

Practical Directions.—Fasten a sheet of paper on the drawing-board, and on it place the square of glass plate.

The glass should be at least $\frac{1}{4}$ of an inch thick.

Stick two pins, *F* and *E*, vertically into the board so that

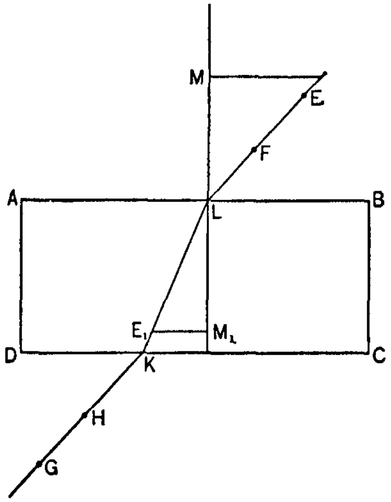

FIG. 11.

the line joining them makes an angle of about 30° with one edge of the plate.

Now look through the opposite face of the plate, and the refracted images of the two pins can be seen.

Adjust the position of the eye till the two images appear in a straight line.

Mark the direction of this line by sticking two other pins, *G* and *H*, so that these two and the refracted images of the other two are in one straight line.

Mark the position of the glass on the paper, and remove it.

Join FE, and produce to cut the line which marks the edge AB of the glass in L.

Join GH, and produce to meet the edge CD of the glass in K.

Join LK. Then the ray of light falling on the glass at L, along the line EF, is refracted through the glass along the line KL, and emerges along the line GH.

Measure off LE, 10 cm., and by means of a set-square erect a perpendicular from E on the normal MFM_1.

Then $$\sin \phi = \frac{EM}{10}.$$

Produce if necessary LK to E_1 till LE_1 is 10 cm. long, and erect a perpendicular E_1M_1 on MFM_1.

$$\sin \phi_1 = \frac{E_1M_1}{10}.$$

Measure EM and E_1M_1.

$$\mu = \frac{EM}{E_1M_1}.$$

Repeat the observations for three different values of ϕ.

Example.—Enter results thus:

EM.	E_1M_1.	μ.
5.67	3.72	1 51
6.47	4.26	1.52
9.08	6.01	1.51
Mean value..............................		1.515

Show complete diagram in each case.

Blank to be filled in by student.

EM.	E_1M_1.	μ.
Mean value of μ........................		

---

16. (I) TO VERIFY THE LAW THAT WHEN A RAY OF LIGHT IS REFRACTED THROUGH A PRISM, THE ANGLE OF INCIDENCE PLUS THE ANGLE OF EMERGENCE IS EQUAL TO THE DEVIATION PLUS THE ANGLE OF THE PRISM.

(II) TO FIND THE REFRACTIVE INDEX OF THE PRISM.

References.—Barker, p. 431; Carhart, p. 267; Hastings and Beach, p. 617; Knott, p. 263; Ames, p. 429.

Apparatus Required.—A prism; a pair of dividers; a centimeter scale; three pins; a set-square.

Theory of Experiment.—If a ray of light fall upon the prism ABC (Fig. 12) from a luminous point P, at the point Q, and be bent through the prism along a direction QR, emerging along RS, PQD is the angle of incidence, and ERS the angle of emergence, where DQ and ER are perpendicular respectively to AB and AC.

Denote PQD by ϕ, ERS by ψ, BAC by I, and LFS by δ. Since PQ and RS meet in F, SFL is the deviation,

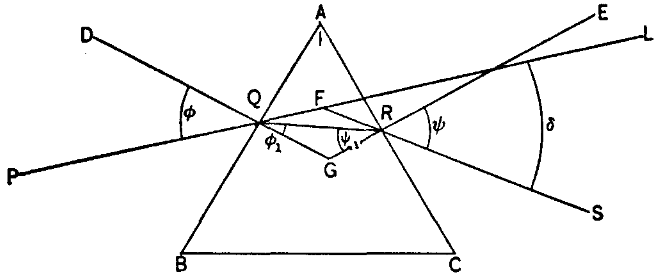

FIG. 12.

and
$$QFS = 180 - \delta,$$
$$QGR = 180 - I,$$
$$FQG = \phi,$$
$$FRG = \psi.$$

Hence $180° - I + 180° - \delta + \phi + \psi = 360°,$

and therefore $\phi + \psi = \delta + I.$ . . . . . . (1)

If the incident ray fall so that QR is parallel to BC, then $\phi = \psi,$

and in that case $2\phi = \delta + I,$ or $\phi = \frac{\delta + I}{2}.$

Now if μ be the refraction index of the prism,

$$\mu = \frac{\sin \phi}{\sin RQG} = \frac{\sin \phi}{\sin \phi_1}.$$

When $\phi = \psi$, $RQG = QRG$, or $\phi_1 = \psi_1$.

Now since $\phi_1 + \psi_1 + 180 - i = 180°$, therefore

$$2\phi_1 = I, \quad \text{or} \quad \phi_1 = \frac{I}{2}.$$

Hence
$$\mu = \frac{\sin \frac{\delta + I}{2}}{\sin \frac{I}{2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

It may be shown geometrically that when $\phi = \psi$, the deviation, δ, is a minimum.

Practical Directions.—(1) Describe a circle with a radius of 10 cm., Fig. 13.

Place the prism with its edge at A, the centre of the circle. Stick a pin vertically at a point P, such that the angle which PA makes with the face AB is less than a right angle. Observe the direction of the refracted image along the line AR.

Stick a pin at R, in such a position that the image of the pin at P is in a line with the edge of the prism and R.

Draw DA perpendicular to the face AB, and EA perpendicular to the face AC. Join PA and RA, and produce PA till it cuts the circle in F.

Then
$$PAD = \phi,$$
$$EAR = \psi,$$
$$FAR = \delta.$$

Draw PO perpendicular to DA, EN and FM to AR.

Then,
$$\sin \phi = \frac{PO}{r},$$

$$\sin \psi = \frac{EN}{r},$$

$$\sin \delta = \frac{FM}{r},$$

where r is the radius of the circle.

Measure PO, EN, FM, and calculate the values of angles ϕ, ψ, and δ from the sines.

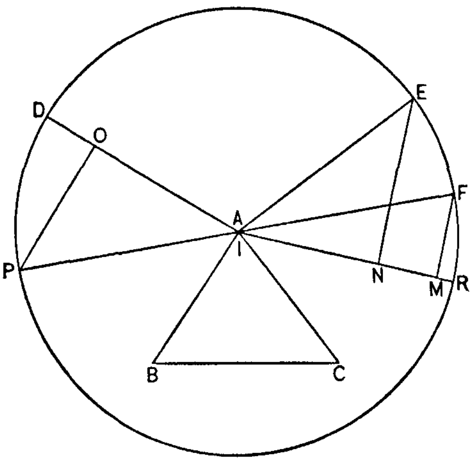

FIG. 13.

Measure I, the angle of the prism, by the method of Experiment 14.

Substitute the values in formula (1).

(II) Now adjust for minimum deviation. To accomplish this turn the prism around, with the edge at the centre as axis, and observe whether the deviation increases or decreases by observing whether the direction, AR, of the refracted ray makes a larger or smaller angle with the direction, AF, of the incident ray. If the angle be decreasing, continue to turn the prism in the same direction; if not, reverse the process, when, if the position be not already one of minimum deviation, the angle will decrease.

If the process be continued, it will be found that angle FAR reaches a minimum value and then increases no matter which way the prism is turned.

Adjust for the exact position of minimum deviation and measure δ as before.

Calculate μ from formula (2).

Example.—Enter results thus:

Show complete diagram in each case.

I.

$PQ = 7.7$

$\sin \phi = .77, \quad \phi = 50° \, 24'$

$EN = 9.85$

$\sin \psi = .985, \quad \psi = 80° \, 06'$

$MF = 9.42$

$\sin \delta = .942, \quad \delta = 70° \, 24'$

$I = 60° \, 00'$

$\phi + \psi = 130° \, 30'$

$\delta + I = 130° \, 24'$

Difference $6'$

II.

$FM = 6.18$

$\sin \delta = .618$

$\delta = 38° \, 12'$

$$\mu = \frac{\sin \frac{\delta + I}{2}}{\sin \frac{I}{2}}$$

$$= \frac{\sin 49° \, 6'}{\sin 30°}$$

$$= 1.51$$

Blank to be filled in by student.

$PQ =$

$\sin \phi = \qquad \phi =$

$EN =$

$\sin \psi = \qquad \psi =$

$MF =$

$\sin \delta = \qquad \delta =$

$I =$

$\phi + \psi =$

$\delta + I =$

Difference

$FM =$

$\sin \delta =$

$\delta =$

$\mu =$

$=$

$=$

17. TO MEASURE THE RADIUS OF CURVATURE OF A SPHERICAL SURFACE BY MEANS OF A SPHEROMETER.

References.—Watson, p. 459; Ames, p. 413; Nichols and Franklin, vol. III. p. 32; Anthony and Brackett, p. 8; Barker, p. 419; Carhart, pt. I, p. 249.

Apparatus Required.—A spherometer; a spherical surface; a centimeter scale.

Theory of Experiment.—The spherometer consists of a metal-tripod with three equidistant legs. Through the centre of the collar to which the legs are attached is a fine screw carrying a graduated disk, which moves quite near a graduated upright scale attached to one of the legs. The instrument is first set on a plane glass surface and the centre screw turned till the point just touches the surface of the glass. It is then transferred to the spherical surface, and the centre screw turned until it again touches the surface. If δ be the difference of the two readings, and l the distance between the legs of the spherometer, then the radius of the spherical surface is given by the equation

$$r = \frac{l^2}{6\delta} + \frac{\delta}{2}.$$

This may be shown thus:

Let $ABDC$ be the spherical surface (Fig. 14), and D the point where the screw of the spherometer touches the surface when the three legs are also touching it.

Then AB is the diameter of the circle passing through three points where the legs rest on the surface.

If these three points be joined on the plane of this circle, an equilateral triangle would be formed.

Denoting the distance between the legs by l, we have geometrically

$$a^2 = \frac{l^2}{3}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where a is the radius of the small circle.

Now $$(2r - \delta)\delta = a^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

since DBC is a semicircle, where δ is the distance from the point D on the surface to the plane of the feet of the

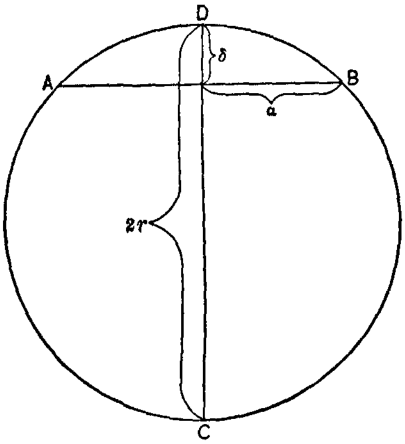

FIG. 14.

spherometer, that is, the distance through which the point must be moved from its first to its second position.

Hence, combining (1) and (2) and solving for r, we get

$$r = \frac{l^2}{6\delta} + \frac{\delta}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Practical Directions.—The screw of the spherometer has usually a pitch of $\frac{1}{2}$ mm., and the upright scale is similarly divided.

The graduated disk is also divided so as to give exact fractions of a turn.

Place the spherometer on the plane glass surface provided for the purpose, and turn the screw until it just touches the surface of the glass.

Read the upright scale and also the disk.

Place the spherometer upon the spherical surface and turn the screw until it again just touches the surface.

Read the upright scale and the disk.

If the graduated disk be divided into 100 parts, divide its reading by 2, and add to the reading, expressed in millimeters, of the upright scale.

If, however, the graduated disk be divided into 50 parts, add at once the reading as the decimal of a millimeter.

The difference between the first and second readings gives the value of δ.

As the value of δ is very small, great care must be taken with the observations in order to secure accuracy.

Take at least six readings in each case.

Express the mean difference in centimeters.

Measure the distance between the legs in centimeters and substitute in the formula.

Example.—Enter results thus:

Reading on Plane Surface.	Reading on Spherical Surface.	δ (mm.)	l (cm.)	r (cm.)
9.95	10.23	.28		
9 96	10.23	.27		
9.94	10.24	.30	8.5	
9.95	10.22	.27		72.45
9.93	10.23	.29		
9.96	10.24	.28		
Mean value...........		.282 mm.		

Blank to be filled in by student.

Reading on Plane Surface.	Reading on Spherical Surface.	δ (mm.)	l (cm.)	r (cm.)
Mean value...........				

18. TO DETERMINE THE RADIUS OF CURVATURE OF A CONCAVE MIRROR BY REFLECTION.

References.—Knott, pt. II. p. 256; Nichols and Franklin, vol. III. p. 32; Ames, p. 413; Hastings and Beach, p. 613; Anthony and Brackett, p. 408; Watson, p. 459; Barker, p. 419; Carhart, pt. I. p. 249.

Apparatus Required.—A concave mirror; a clip-stand; a pin; a centimeter scale; a spherometer.

Theory of Experiment.—If an object be held in front of a concave mirror beyond its geometrical centre, an inverted

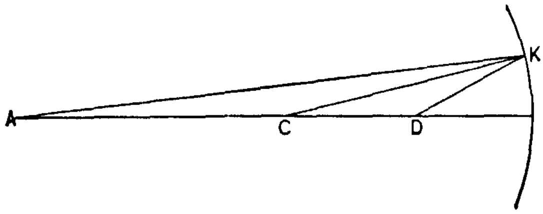

Fig. 15.

image of the object will be seen between the object and the mirror.

Thus if the object be held at A (Fig. 15), and C be the geometrical centre, the image will be seen at a point D, when the angle $AKC = CKD$.

If now the object be moved up to the centre, C, the direct and reflected rays will have the same path along CK.

The image will therefore coincide with the object.

If, therefore, the object be so placed that the image is seen to coincide with it, the distance of the object from the mirror is the radius of the mirror.

Since f, the focal length, is equal to one-half the radius, it can be obtained directly.

Practical Directions.—Place a pin vertically in a clip in front of the mirror.

Adjust its position so that an inverted image of the pin can be seen between the pin and the mirror.

Move the clip toward the mirror, and adjust until the point of the image appears to coincide with the point of the pin. To determine the exact position of coincidence, let the pin and the image slightly overlap and then move the eye

back and forth so that they can be seen from different points of the mirror. When the point of exact coincidence is found, the pin and image will continue to occupy the same relative position to each other, no matter at what point of the mirror they may be observed.

Having thus found the point, measure by means of a centimeter scale the distance of the pin from the mirror.

Repeat the operation several times.

The mean of the observations may be taken as the radius.

Verify your results by a spherometer.

Example.—Enter results thus:

Observation.	r
1st	72.23
2d	72.31
3d	72.25
4th	72.20
5th	72.18
6th	72.24
Mean value	72.23

r by spherometer....... 72.35

Blank to be filled in by student.

Observation.	r
Mean value	

r by spherometer.......

19. TO DETERMINE THE RADIUS OF CURVATURE OF A CONVEX MIRROR.

References.—As in Experiment 18.

Apparatus Required.—A convex mirror, suitably mounted; two clamp-stands holding small upright rods; a tape measure; a centimeter scale; a telescope.

Theory of Experiment.—If OO_1O_2 (Fig. 16) be the axis of a convex mirror, BB_1; A and A_1 two objects situated so

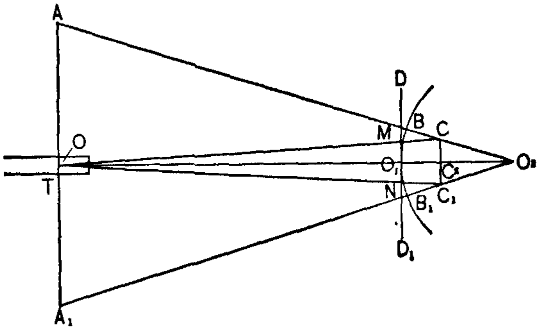

FIG. 16.

that AO is equal to A_1O, and AA_1 at right angles to OO_1, C and C_1 the positions of the image of A and A_1 as seen in the mirror, O_2 being the centre of the spherical surface; then

$$\frac{1}{AB} - \frac{1}{BC} = -\frac{2}{O_2B}, \quad . \quad . \quad . \quad . \quad (1)$$

or

$$\frac{1}{u} - \frac{1}{v} = -\frac{2}{r},$$

where u, v, and r are respectively equal to AB, BC, and O_2B.

Hence

$$v = \frac{ur}{2u + r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Denote CC_1 by x_1, AA_1 by x, and MN, the intercept

on the tangent to the surface at O_1 made by joining OC and OC_1, by x_2.

Then from similar triangles we have

$$\frac{x}{x_1} = \frac{u + r}{r - v}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and

$$\frac{x_1}{x_2} = \frac{OO_1 + O_1C_2}{OO_1},$$

or

$$\frac{x_1}{x_2} = \frac{u + v}{u}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

since, OO_1 being large as compared with AA_1, OO_1 is approximately equal to AB.

Combining (3) and (4), substituting for v the value found in (1), and solving for r, we obtain

$$r = \frac{2ux_2}{x - 2x_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In practice the distance OO_1 may be substituted for u for the reason given above.

The measurement of MN (x_2) requires the use of a telescope.

Practical Directions.—Fix the mirror in the clamp provided and in an upright position.

Place the telescope at a distance of two or three meters from the mirror and adjust its direction and height until the axis of the telescope is in line with the axis of the mirror.

Place the clamp-stands with the upright rods in the positions corresponding to A and A_1 (Fig. 16), the line joining them passing through the object-glass of the telescope and being perpendicular to its axis. AA_1 should be from 40 to 70 cm.

The telescope can now be focussed on the images of the upright rods seen in the mirror.

To obtain the intercept at the surface of the mirror corresponding to MN or x_2, fasten a centimeter scale across the

face of the mirror in a position corresponding to DD_1 in the figure. The upper edge of the scale should approximately bisect the mirror.

By slightly altering the focus of the telescope both the scale and image can be seen and the distance between the images as seen on the scale observed.

Read this distance, x_2.

Measure the distance, x, between A and A_1.

Measure the distance between the object-glass of the telescope and the surface of the mirror, u.

Substitute in formula (5) and calculate r.

Repeat the observations, changing the positions of the telescope and upright rods each time.

Verify your results by the spherometer.

Example.—Enter results thus:

Observation.	u	x	x_2	r
(1)	300	60	6.22	78.5
(2)	250	55.5	6.65	78.7
(3)	275	48.7	5.40	78.4
Mean value of r....................................				78.5
r by spherometer..............................				78.6

Blank to be filled in by student.

Observations.	u	x	x_2	r
Mean value of r..............				
r by spherometer...				

20. TO DETERMINE THE FOCAL LENGTH OF A CONVEX LENS BY PARALLEL RAYS. METHOD I.

References.—Barker, p. 435; Watson, p. 480; Carhart, pt. I. p. 273; Hastings and Beach, p. 619; Nichols and Franklin, vol. III. p. 45; Knott, pt. II. p. 267; Ames, p. 440; *Anthony and Brackett*, p. 412.

Apparatus Required.—An elementary optical bench provided with ground-glass screen; several convex lenses; a telescope.

Theory of Experiment.—(*a*) If f be the focal length of the lens, u and v the respective distances of the object and image from the surface of the lens, then we have, from the law of convex lenses,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If the incident rays be parallel,

$$\frac{1}{u} = 0,$$

and hence

$$v = f. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

It is only necessary, therefore, to observe v.

(*b*) Another method involving the same principle is as follows: If a telescope be carefully focussed on a very distant object, and afterwards it be used to view a near object through a convex lens, the distance between the lens and the object will be equal *to the focal length of the former*, when a sharp image of the object is seen in the telescope.

For, only parallel rays will come to focus in the telescope, and the rays after traversing a lens from a given object are parallel when the distance between lens and object is equal to

the focal length of the lens. In this as in the preceding case only one observation is necessary.

Practical Directions.—(*a*) Select a lens having a focal length not greater than the length of the bench.

Mount it on its stand with its axis along the bench.

Mount the ground-glass screen behind it, with the plane of the screen at right angles to the direction of the bench. It is important to have the lens and ground-glass screen occupy the same position in relation to the indexes which are carried by them.

Point the apparatus to an open window so that an image of a distant object, such as a church-spire, may be obtained on the screen.

Slide the lens along the bench until a clearly defined image of the object is obtained.

Read the distance between the indexes carried by the lens and screen.

This distance is the focal length.

Repeat the observation three times for each lens and take the mean of the results.

The image is best viewed from behind the screen.

(*b*) Select an ordinary reading telescope and focus it out of doors on a very distant object.

Place the telescope on the optical bench, close up to the lens in question, so as to look through it in the direction of the bench.

Mount a piece of white printed paper on one of the bench-stands, so that the plane of the paper is the same as that of the index carried by the stand.

Illuminate the paper by a strong light which is shaded from the lens.

Move the paper along the bench until a clear image of the printing is seen in the telescope.

Read the distance between the lens and object.

Make three such readings, and take their mean for the value of f.

Example.—Enter results thus:

Method *a*.			Method *b*.		
Lens.	*f*.	Mean Value for *f*.	Lens.	*f*.	Mean Value for *f*.
A	12.0		A	12.1	
	12.2			12.0	
	12.3	12.25		12.2	12.10
B	17.2		B	17.3	
	17.3			17.4	
	17.4	17.30		17.3	17.63
C	29.7		C	29.8	
	29.9			29.8	
	30.0	29.87		29.9	29.83

Blank to be filled in by student.

Method *a*.			Method *b*.		
Lens.	*f*.	Mean value for *f*.	Lens.	*f*.	Mean value for *f*.

21. TO FIND THE FOCAL LENGTH OF A CONVEX LENS BY THE DISTANCES OF THE OBJECT AND IMAGE FROM THE LENS. METHOD II.

References.—As in Method I.

Apparatus Required.—In addition to that of Method I, a lamp and fine wire grating or other suitable object for illumination will be required.

Theory of Experiment.—As before, u and v being the respective distances of the object and image from the lens, we have

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f},$$

from which f may be readily calculated, if u and v be observed.

Practical Directions.—Mount on one of the stands the fine wire grating, with the plane at right angles to the bench. Cover the grating with a large sheet of paper having a small hole near the centre.

Mount the lens on the second or middle stand, so that its axis lies along the bench in a horizontal line with the centre of the hole.

The third stand carries the ground-glass screen, mounted at right angles to the bench, so as to receive the image of the wire gauze.

The object, lens, and ground-glass screen should occupy the same positions in relation to the indexes carried by them.

Place the light directly behind the hole in the paper, and as close to it as possible.

Adjust the positions of the lens and screen along the bench until a clearly defined image of the illuminated object is obtained.

If the focal length of the lens be less than one-fourth the available length of the bench, an image of the illuminated wire grating can in this manner be readily obtained.

The image is best observed from behind the ground-glass. Read the position of the indexes carried by the wire screen, lens, and ground-glass screen.

The adjustment of the position of the lens should be made three times, and a mean of the readings taken for u and v.

Calculate from u and v, the distances of the object and image from the lens, the value of f, using formula (1).

Repeat the observations three times, and take the mean value of f.

If there be too much glare from the light behind the wire grating, it will be well to cover it with thin white paper.

The experiment must be performed in a darkened room.

Example.—Enter results thus:

LENS A.

u.	Mean u.	v.	Mean v.	f.	Mean f.
30. 29.8 30.1	29.96	20. 20.2 19.9	20.03	12	
40. 40.2 40.4	40.2	17.1 16.9 16.7	16.9	11.9	
36. 35.8 36.2	36.0	18.2 18.4 18.0	18.2	12.1	12

Blank to be filled in by student.

u.	Mean u.	v.	Mean v.	f.	Mean f.

22. TO FIND THE FOCAL LENGTH OF A CONVEX LENS BY CHANGING THE POSITION OF THE LENS. METHOD III.

References.—Same as Method I(a).

Apparatus Required—As in Method II.

Theory of Experiment.—If the distance between object and screen be more than four times the focal length of the lens, the lens will have two positions where a clearly defined image of the object will be obtained on the ground-glass screen.

Let the distance between the object and screen be l, that between the two positions of the lens a, and, as before, f the focal length. Then we have

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}, \text{ for the first position,}$$

and

$$\frac{1}{u_1} + \frac{1}{v_1} = \frac{1}{f}, \text{ for the second.}$$

Further, it is clear, since l is constant, that $u = v_1$ and $v = u_1$; that is, the lens will be at the same distance from the ground-glass screen in the second case as it was from the object in the first.

Hence we have

$$u + v = l, \quad u_1 - u = a, \quad u_1 = v,$$

and therefore

$$u = \frac{l - a}{2}, \qquad v = \frac{l + a}{2}.$$

Substituting these values of u and v in equation (1), we obtain

$$f = \frac{l^2 - a^2}{4l}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The above relation is independent of the distances between the surface of the lens and the object and image, which distances are much more difficult to measure accurately than the distance a.

For accurate work it must, however, be remembered that $u + v$ is not equal to l, as the distances u and v are not strictly measured from one point, but from the principal points of the lens. These principal points are one-third the thickness of the lens apart, and therefore the formula for thick lenses or combinations would have to be corrected. This correction, however, for an ordinary long-focus lens of 20 cm. or so, is unimportant.

Practical Directions.—The adjustments are as in Method II. Bring the lens up to the ground-glass screen until a sharp image of the object is obtained. If this image is inconveniently small, the ground-glass screen should be moved nearer the object, and the lens refocussed.

Read the index carried by the lens.

Move the lens into the other position near the object, the ground-glass screen remaining fixed, until a sharp image of the object is again obtained in the ground glass.

Read the lens index again.

The difference between the two readings gives a.

Read the indexes carried by the object and ground-glass screen. Hence the distance l.

The observations for a should be made by adjusting the lens three times in each position, and a mean taken.

A more accurate way of determining the coincidence of the image with the ground-glass screen is to substitute for it a wire gauze with its wires inclined at an angle of 45° to the wires of the gauze used for the object.

Focus a small reading-telescope of high power on the wires of the second screen.

Move the lens until, on looking through the telescope, the wires of the illuminated screen are seen in focus with the other. No confusion of screen and image can arise if the two be inclined to each other as suggested.

It is important that the image screen should not be moved after focussing the telescope.

A good plan is to use a powerful magnifier like the positive eyepiece of a telescope, and mount it on the second screen-stand so as to move with it. Most precise coincidence can be obtained in this way.

Example.—Enter results thus:

l	a	Mean a	f
100	72.1 72.0 72.2	72.15	12.0

Blank to be filled in by student.

l	a	Mean a	f

23. TO FIND THE FOCAL LENGTH OF A CONVEX LENS FROM THE SIZE OF THE MAGNIFIED IMAGES. METHOD IV.*

References.—As in Method II.

Apparatus Required.—A transparent scale, finely divided (an ordinary opal-glass scale answers well); a large white-paper screen; a lens of rather short focus; a pair of dividers; a lamp and optical bench.

Theory of Experiment.—Let l be the length of a division of the transparent scale which is used as an object. Let L be the length of a division of the magnified image. Let v be the distance of the screen from the centre of the lens when a sharp image occurs on it.

Then we have the ordinary relation

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where u is the distance from the illuminated scale to the lens.

We also have the relation

$$\frac{L}{v}=\frac{l}{u}, \quad \text{or} \quad \frac{1}{u}=\frac{L}{lv}.$$

Hence by substituting in (1) we obtain

$$f=v\frac{l}{L+l}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

which is the relation required.

Practical Directions.—As in previous methods, the axis of the lens must lie along the bench and in the same horizontal as the centre of the illuminated scale.

---

*This method is applicable to thick lenses or combinations when the distance between the principal points cannot be neglected.

See that the index carried by the lens is in the same plane as the *centre* of the lens, and that the index of the screen lies in the plane of the screen.

Place the lens at a little greater distance than its focal length from the scale.

Move up the white screen until a sharply defined image of the scale division is obtained.

Measure to $\frac{1}{10}$ mm. the length of as great a number of magnified divisions as are obtained on the screen.

Read the distance between the lens and surface of the screen.

Calculate the value of a single magnified division in terms of the object scale.

Repeat the observations three times, and take the mean value of f calculated from formula (2).

Example.—Enter results thus:

v	l	L	f
85.0	1	3.15	20.40
63.5	1	2.12	20.30
48.7	1	1.4	20.30
Mean value of f			20.33

Blank to be filled in by student.

v	l	L	f
Mean value of f........			

24. TO DETERMINE THE FOCAL LENGTH OF A CONCAVE LENS BY THE DIVERGENCE OF THE REFRACTED RAYS. METHOD I.

References.— Watson, p. 481; Knott, pt. II. p. 269; Hastings and Beach, p. 619; Nichols and Franklin, p. 45; Ames, p. 440; Anthony and Brackett, p. 412.

Apparatus Required.—An elementary optical bench; a moderately long focus concave lens; a lamp; a ground-glass screen; a black-paper screen with two small apertures not greater than the width of the lens apart; a pair of dividers; a centimeter scale.

Theory of Experiment.—Let u and v be the respective distances of the source of light and the virtual image from the face of the concave lens AB.

Then we have the ordinary formula for f, the focal length of concave lenses,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

u and v being in this case both on the same side of the lens. Of these values u can be measured directly.

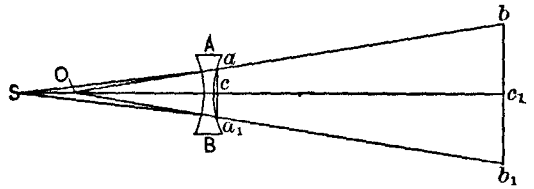

FIG. 17.

If now the face of the lens be covered with a sheet of black paper with two apertures, a, a_1, the light passing

through these apertures will give two bright patches of light, b, b_1, on a screen placed to receive them. Then

$$\frac{aa_1}{bb_1} = \frac{v}{v + cc_1},$$

or

$$v = \frac{aa_1 \cdot cc_1}{bb_1 - aa_1}.$$

Since $a\ a_1$, $b\ b_1$, $c\ c_1$ can be measured, v can be calculated.

Substituting in formula (1), f can be calculated.

Practical Directions.—Mount the lens in the middle stand with its axis horizontal and along the bench.

Make two fine holes in the black paper not more than 2 or 3 cm. apart, and fix it to the surface of the lens so that the apertures are central with regard to it.

Mount the lamp (an incandescent lamp, with the filament edge on, answers well) on one of the outside stands.

Adjust the height of the lamp so that the centre of the filament is in the same horizontal as the axis of the lens.

Mount the ground-glass screen behind the lens at right angles to the bench, so as to receive the divergent rays of light. Move the lens along the bench until a considerable divergence is obtained.

Measure carefully by means of the dividers and scale the distance between the centres of the bright spots on the ground-glass screen, and also to the tenth of a millimeter the distance between the apertures.

Read on the bench the distances of the lamp and ground-glass screen from the surface of the lens next the apertures. The relative positions of the lamp, lens, and ground-glass screen to their indexes should be carefully allowed for. The adjustment of the position of the lens should be made three

times, and the mean of the calculated value of v taken. Calculate v from formula (2), and substitute in (1) for f.

Example.—Enter results thus:

u.	aa_1.	bb_1.	cc_1.	v.	f.
77.0	2.0 2 2.0	6.3 6.4 6.3	26.5 26.5 26.5	12.33 12.33 12.32	14.6

Blank to be filled in by student.

u.	aa_1.	bb_1.	cc_1.	v.	f.

25. TO DETERMINE THE FOCAL LENGTH OF A CONCAVE LENS BY AN AUXILIARY CONVEX LENS. METHOD II.

References.—As in previous experiment.

Apparatus Required.—The same as in Method I, except the black-paper screen, and, in addition, a suitable convex lens of known focal length.

Theory of Experiment.—A more accurate method than the preceding is obtained by making a combination with a convex lens of sufficient power to render the combination slightly convex. Suppose *AB* (Fig. 18) a concave lens, and *CD* a convex lens of known focal length, so that the two together make a convex system.

Consider the light traversing the concave lens from a bright object at O. It is refracted so as to form a virtual image at a, and we have therefore

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $aM = v$, and $OM = u$, and f is the focal length of the concave lens.

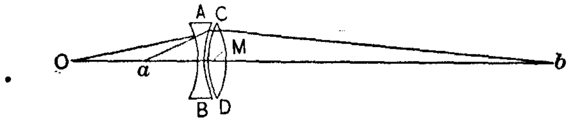

FIG. 18.

The rays are again bent from the path aC, and refracted to a focus point b by the convex lens.

Then as far as the convex lens is concerned the source of light is at a.

We have, therefore,

$$\frac{1}{v} + \frac{1}{v_1} = \frac{1}{f_1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where $aM = v$, and $Mb = v_1$, and f_1 is the focal length of the convex lens.

Let F be the focal length of the combination.

Then the light from O is brought to a focus at b by the combination of the two lenses.

It follows, therefore, that

$$\frac{1}{u} + \frac{1}{v_1} = \frac{1}{F}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Hence, combining (1), (2), and (3), we obtain the relation

$$\frac{1}{f} = \frac{1}{f_1} - \frac{1}{F},$$

or

$$f = \frac{Ff_1}{F - f_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

F can be calculated from formula (3), and f from formula (4), f_1 being known or found separately.

Practical Directions.—The adjustments and observations are the same both for the convex lens and the combination, as in the case of convex lenses.

It is readily seen by inspection of formula (4) that some care is necessary in choosing the auxiliary lens. For if $F - f_1$ be small, small errors in measuring them, unless the errors happen to be in the same direction, would result in a large error in f. The convex lens should therefore be chosen so as to make the difference $F - f_1$ as large as possible, or the combination should be equivalent to a lens with very slight convexity, so that f is very nearly equal to f_1.

Example.—Enter results thus:

Observations for f_1.		Convex Lens.	Observations for F.			f.
u.	v.	f_1.	u.	v_1.	F.	
30.0	20.0	12	120	165.9	69.6	14.5
29.8	20.2	11.9	120.5	165.4	69.7	14.5
31.1	19.9	12.1	119.8	166.1	69.6	14.5

Blank to be filled in by student.

Observations for f_1.		Convex Lens.	Observations for F.			f.
u.	v.	f_1.	u.	v_1.	F.	

26. (1) TO CONSTRUCT A MICROSCOPE.
(2) TO CONSTRUCT A TELESCOPE.

References.—Anthony and Brackett, p. 425; Ames, pp. 450–452; Hastings and Beach, pp. 631–637; Barker, pp. 456–471; Knott, pt. II. p. 284; Nichols and Franklin, vol. III. pp 57–71; Watson, pp. 489–493.

Apparatus Required.—Three short-focus lenses and one long-focus lens, suitably mounted; a centimeter scale; a piece of wire gauze in a clamp-stand.

Theory of Experiment.—(1) *The Microscope.*—If an object AB be placed in front of a short-focus lens PQ so as to be just beyond its principal focus, a real inverted and slightly magnified image of the object will be formed on the opposite side of the lens from AB as A_1B_1.

If now a second lens, MN, be placed so that the image A_1B_1 is just inside its principal focus, a vertical and magnified image of A_1B_1 will be produced on the same side of MN as A_1B_1, see $A_{11}B_{11}$, Fig. 19.

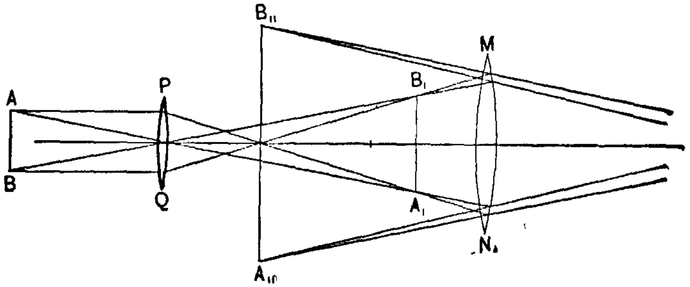

FIG. 19.

The lens MN with respect to the image A_1B_1 forms a simple microscope. The two lenses with respect to the object AB form a compound microscope.

PQ is called the object-glass, MN the eyepiece.

(2) *The Telescope.*—The telescope is constructed on the same principal as the miscroscope. In the case of the telescope, however, the object-glass is a long-focus lens and forms a diminished image of a *distant* object instead of a magnified image of a *near* object. As in the case of the miscroscope, the eyepiece is used to magnify the image obtained by means of the object-glass.

Practical Directions.—(1) *The Miscroscope.*—A centimeter scale, held vertically, makes a suitable object.

In front of it place one of the short-focus lenses at a distance a little greater than its focal length. A suitable position can be found by placing the lens quite near the object and then moving it gradually away until a real inverted image is seen between the eye and the lens.

To find the exact position of the image so that the eyepiece can be adjusted, a piece of wire gauze, mounted on a stand, can be used. Adjust the position of the gauze until it appears to coincide with the image.

The point of exact coincidence can be obtained by moving the eye back and forth in a plane parallel to the gauze and adjusting until the gauze and image continue to occupy the same relative position from whatever point they be viewed.

Take another of the short-focus lenses and focus it upon the edge of the gauze coincident with the image.

Remove the gauze, and a magnified image of the scale will be seen.

Measure a and b, the distances from the object to the object-glass and from the object-glass to the eyepiece respectively.

Repeat the observations three times, changing the distance a in each case.

(2) The adjustments for the telescope are exactly the

same as for the miscroscope, the only difference being that the object-glass is a long-focus lens and its distance from the object much greater.

Example.—Enter results thus:

Miscroscope.		Telescope.	
a	*b*	*a*	*b*
12.5 15.5 19.5	23.0 26.5 30.5	74.5 168.4 121.0	98.3 183.0 141.5

Blank to be filled in by student.

Miscroscope.		Telescope.	
a	*b*	*a*	*b*

27. TO DETERMINE THE MAGNIFYING POWER OF A MICROSCOPE.

References.—As in Experiment 26.

Apparatus Required.—A compound microscope; two millimeter scales.

Theory of Experiment.—The magnifying power of a microscope is the ratio of the angle subtended at the eye by the image to that subtended by the object, both being at the distance of distinct vision, about 25 cm. If, therefore, a microscope be focussed on a finely divided scale and the image be observed with one eye, while the other eye looks at

a second similarly divided scale, 25 cm. distant and so placed that the image of the first appears to coincide with it, the number of divisions of the second scale covered by one of the magnified divisions of the image gives the magnifying power.

Similarly if the magnifying powers of each of the lenses be observed, their product will be the magnifying power of the microscope.

Practical Directions.—(1) Focus the microscope upon a millimeter scale.

Place another millimeter scale at the side of the instrument at a distance of about 25 cm.

Looking through the microscope with one eye, adjust the position of the second scale until the image of the first as seen in the microscope appears to coincide with the second scale as seen by the other eye along the side of the microscope. Count the number of scale divisions of the second scale covered by as many of the magnified divisions of the image as can be accurately observed.

Denoting the number of divisions of scale by a, the corresponding divisions of the image by b, and the magnifying power by M, then

$$M = \frac{a}{b}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Repeat the observations several times and take a mean of the results.

(2) The magnifying powers of the eyepiece and the object-glass may be found separately by a similar method, if the microscope contain in the eyepiece a micrometer scale the value of the divisions of which are known.

Focus the microscope on the millimeter scale and note the number of divisions of the image, which is magnified by both the eyepiece and the object-glass, covered by a number of divisions of the micrometer scale, which is magnified by the eyepiece only.

The ratio of the two, expressed in the same units, gives the magnifying power of the object-glass.

Thus, if we denote the magnifying power of the object-glass by m, the divisions of the scale by b_1, the corresponding micrometer divisions by c, and the constant, required to reduce micrometer divisions to scale divisions by δ, then

$$m = \frac{c\delta}{b_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Observe now, with one eye along the side of the microscope, the number of divisions of the scale covered by a definite number of divisions of the micrometer scale as seen by the other eye through the microscope.

Since the micrometer scale is magnified by the eyepiece only, the ratio of these two, when expressed in the same units, gives the magnifying power of the eyepiece.

If b_2 be the scale divisions, c_1 the corresponding micrometer divisions, and m_1 the magnifying power of the eyepiece, then

$$m_1 = \frac{b_2}{c_1\delta}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The product, $m \cdot m_1$, gives M.

Repeat the observations several times for both eyepiece and object-glass.

Example.—Enter results thus:

FIRST METHOD.

a	b	M
42	2	21
63	3	21
42	2	21
Mean value of M.......		21

SECOND METHOD.

δ	Object-glass.			Eyepiece.			M
	b_1	c	m	b_2	c_1	m_1	
.63	3	34	7.14	20	11	2.88	20.57
	2	23	7.24	10	6	2.88	20.80
	3	34	7.14	20	11	2.88	20.57
Mean value of M....................................							20.64

Blanks to be filled in by student.

FIRST METHOD.

a	b	M
Mean value of M........		

SECOND METHOD.

δ	Object-glass.			Eyepiece.			M
	b_1	c	m	b_2	c_1	m_1	
Mean value of M..							

28. TO DETERMINE THE MAGNIFYING POWER OF A TELESCOPE.

References.—As in Experiment 26.

Apparatus Required.—A white paper scale about 60 cm. long; two strips of white paper; a telescope; a tape measure.

Theory of Experiment.—The magnifying power of a telescope is the ratio of the angle subtended at the eye by the image in the telescope to the angle subtended by the object, the telescope being so focussed that the object and image are at the same distance from the eye. The angles subtended at the eye by the object and image being very small, this ratio will be the same as the ratio of the magnitudes of the image and object, their positions being as stated above.

Hence, if a telescope be focussed on a graduated scale or other distant object and so adjusted that the object and image are at the same distance, D, from the eye, the magnifying power of the telescope for the distance D is given by the equation

$$M = \frac{a_1}{a},$$

where a is the number of image divisions covering a_1 divisions of the scale.

Practical Directions.—Fasten upon the wall of the laboratory in a vertical position the centimeter scale. At equal distances from the ends of the scale, and at right angles to it, fasten the two strips of paper, the middle of the paper being in the axis of the scale in each case. The distance between the strips of paper should be about 75 cm.

Taking the telescope 3 or 4 meters away, focus it upon the scale.

Looking through the telescope with one eye and observing the unmagnified scale with the other, the image will appear projected against the scale.

Adjust the position of the eyepiece until the image occupies the same position as the scale. If the eyepiece has been focussed on the cross-hairs, it will be necessary to pull it out slightly.

The exact position of coincidence of image and scale can be determined as in previous experiments by adjusting the eyepiece until the scale and image continue to occupy the same relative position when the eyes are moved back and forth across the field.

Having found the position of coincidence, read the number of image divisions, a, covered by the distance between the two strips of white paper.

Repeat the observations several times.

Measure the distance a_1 between the strips of paper.

Measure the distance D.

Calculate M.

Repeat the observation three times for different distances of telescope and object.

Example.—Enter results thus:

D (meters).	a_1	Readings for a.	Mean a.	M
4.35	75	5.1 5.0 4.8	4.97	15.1
5.75	75	5.2 5.2 5.3	5.23	14.3
7.28	75	5.7 5.7 5.8	5.73	12.5

Blank to be filled in by student.

D (meters).	a_1	Readings for a.	Mean a.	M

29. THE SPECTROSCOPE.

(1) TO MAP THE SOLAR SPECTRUM AND PLOT THE CALIBRATION CURVE OF THE INSTRUMENT.

(2) TO MAP A BRIGHT-LINE SPECTRUM AND MAKE A TABLE OF CORRESPONDING WAVE-LENGTHS.

References.—Nichols and Franklin, vol. III. p. 76; Carhart, pt. I. p. 293; Anthony and Brackett, pp. 439–444; Ames, pp. 455–467; Barker, pp. 449–462; Hastings and Beach, pp. 704–710; Watson, pp. 514–518; Knott, pt. II. pp. 324–328.

Apparatus Required.—A spectroscope with micrometer scale; Plücker tubes containing H, O, CO, N, etc.; a small induction-coil; a two-volt storage-battery; a map of the solar spectrum; a clamp-stand for Plücker tubes; a striding spirit-level; some small connecting wires.

Theory of Experiment.—For the theory of the experiment read carefully the chapters on *dispersion* and the *solar spectrum* in any of the above references.

Practical Directions.—*Adjustment of the Instrument.*—Focus the telescope by the method of parallax on a distant object. To do this it will generally be necessary to unscrew it from the instrument.

Replace the telescope, and, the prism having been removed, view the slit direct and focus the collimator. This may be done by adjusting the length of the collimator-tube till a sharp image of the slit is seen in the telescope.

Level the collimator and telescope by means of the spirit-level and levelling-screws attached to them. If their vertical height be the same, their axes may be assumed to be in the same plane.

The prism should have its refracting edges at right angles to the above plane. To insure this, level the prism table by the screws provided. Before clamping down the prism, it should be set for minimum deviation, as explained under the spectrometer. (See adjustment for minimum deviation, Experiment 30.)

The instrument should be turned to the window, and, if available, direct sunlight allowed to enter the collimator.

Adjust the width of the slit till sharp narrow images of the dark lines are seen.

If the spectrum be traversed by dark bars at right angles to the solar lines, this is probably due to dust in the slit. This may be removed by introducing the sharpened end of a match into the slit and passing it up and down a few times.

Illuminate the slit in the small tube containing the scale, and clamp the tube in a position such that the whole length of the spectrum is covered by the scale.

Adjust the length of the scale-tube till a well-defined image of the scale is seen in the telescope, after reflection from the near face of the prism.

It may be that the spectroscope is provided with a grad-

uated circle, in which case the scale readings will be read at the index carried by the telescope.

(1) *Mapping the Solar Spectrum.*—With the aid of the map of the solar spectrum observe the position on the scale of all the principal dark lines visible, and draw to scale, on section paper, a map similar to the one below, Fig. 20.

If direct sunlight has not been used, there will probably be no lines visible in the red end before *B*, and none in the violet beyond *G*.

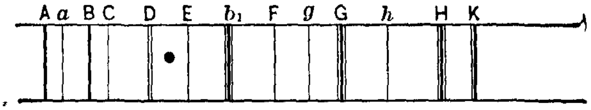

FIG. 20.

Plotting the Calibration Curve of the Instrument.—The following table gives the wave-lengths of the principal dark lines in millionths of a millimeter.

Designation.	Wave-length.
A	760
B	686
C (H)	656
D (Na)	589
E (C and Fe)	527
b (Mg)	518
F (H)	486
G (Fe)	431
H (Ca)	397
K (Ca)	393

With the aid of the above table plot the calibration curve of the instrument.

The scale readings may be plotted as abscissæ to the scale of one scale division equal to one centimeter; and the wave-lengths from the table as ordinates to the scale of fifty equal to two centimeters.

The curve so drawn will be similar to Fig. 21.

(2) *To Measure the Wave-lengths of the Bright Lines in the Spectrum of a Gas.*—Without changing the adjustment of

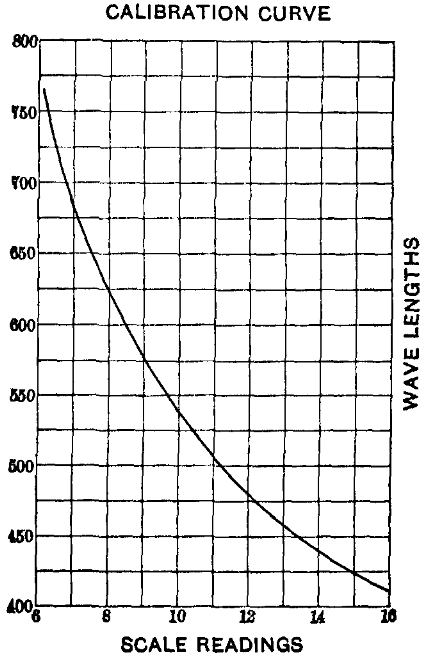

Fig. 21.

the instrument, set up a Plücker tube. If the side-on type is used, have the capillary section vertical and close to the slit.

In the case of the end-on type the capillary section should have its axis in line with the axis of the collimator.

Connect the electrodes of the tube to the secondary terminals of the induction-coil, and the primary of the induction-coil through a switch to the current supply.

For the current supply a portable storage-cell will be found convenient.

See that the contact-breaker works continuously without sparking.

On looking through the telescope the bright lines due to the incandescent gas should be seen. It may be necessary to widen the slit a little.

Identification of the Bright Lines.—Read the positions on the scale of the bright lines, designating them by their color and brightness, and determine their corresponding wave-lengths by the calibration curve.

Precautions.—If the contact-breaker sticks, start it at once, otherwise the coil may be burnt.

Handle the Plücker tubes carefully. Do not alter any of the adjustments between observing the dark and bright lines.

Example.—Enter results thus:

SOLAR SPECTRUM.

Designation of Line.	Scale Reading.	Wave-length from Table.
B	7.16	686
C	7.57	656
D	8.73	589
E	10.30	526
b	10.61	518
F	11.70	486
G	14.60	430

Blank to be filled in by student.

Designation of Line.	Scale Reading.	Wave-length from Table.

Bright-line Spectrum of Oxygen and Hydrogen.—By means of a tube containing oxygen and another containing hydrogen, illuminated by the discharge from the coil, the following bright lines may be observed:

Color of Line.	Scale Reading.	Wave-length from Curve.
	Oxygen	
Red	820	617
"	835	607
Yellow	890	572
Green	935	562
"	10.50	523
"	11.80	488
Blue	12.35	470
	Hydrogen	
Red (*C*)	7.55	659
Blue (*F*)	11.70	486
Violet (*G*)	14.30	430

Blank to be filled in by student.

Color of Line.	Scale Reading.	Wave-length from Curve.

30. TO DETERMINE THE ANGLE OF A PRISM AND TO FIND ITS REFRACTIVE INDEX BY MEANS OF THE SPECTROMETER.

References.—Watson, p. 495; Carhart, pt. I. p. 293.

Apparatus Required.—A spectrometer having a vernier provided for the prism table as well as for the telescope; a prism; a bunsen burner; a spoon of platinum foil for containing the salt for sodium flame; gas-tubing; a spirit-level.

Theory of Experiment.—The Theory of Experiment is the same as that for "The Measurement of the Angle of a Prism by Pin Method," p. 40, and "To Find the Index of Refraction of a Prism," p. 46.

Practical Directions.—The general adjustments are the same as for the spectroscope, p. 84.

To Measure the Angle of the Prism.—(1) *By Moving the Telescope.*—If the adjustment for the parallelism of the incident light has been carefully carried out, no great care need be exercised in centering the angle of the prism in question on its table.

Turn the prism table so that its vernier may be out of range of the moving telescope, and clamp it down.

Turn the prism on its table till the angle to be measured points towards the slit, and clamp it in position.

Illuminate the slit either by the sodium flame or by turning the instrument so that the collimator points to a window.

Turn the telescope to view the reflection of the illuminated slit from one of the faces which bound the angle in question.

Make the slit as narrow as possible, and adjust the position of the telescope by the tangent-screw attached till the vertical cross-wire coincides with the middle of the slit.

Read the position of the telescope on the graduated circle.

Turn the telescope to view the slit from the other face of the angle, reading the position of the telescope as before.

Unclamp the prism table, set it again, and repeat the observations.

The mean difference between the readings on the two sides of the prism is to be taken as twice the angle required.

Difficulty may be experienced at first in finding the reflection of the slit on the faces of the prism by looking through the telescope. It may easily be found, however, with the naked eye, and the telescope then moved up till the image is intercepted.

(2) *By Moving the Prism.*—It will generally be necessary to change the position of the prism on its table so that when the slit is in view on one side, the vernier carried by the prism table is as near as possible to that carried by the telescope.

The observations will be taken in the same way as before, except that the prism table will be moved instead of the telescope, and the readings taken at the vernier carried by the prism table.

The telescope should be displaced a little, and the readings repeated.

To Find the Index of Refraction of the Prism.—It will be necessary in this experiment to have the slit illuminated by the sodium flame.

Remove the prism and turn the telescope to view the slit directly through the collimator.

Set the telescope so that the vertical cross-hair coincides exactly with the middle of the slit, and read the position of the telescope on the graduated circle.

Replace the prism, and turn the telescope so as to view the refracted image of the slit.

To Determine the Minimum Deviation, decrease the angle of incidence by turning the prism table, and follow the refracted ray with the telescope till a point is reached where,

if the prism be turned farther, the refracted ray turns back. Read the position of the telescope.

The difference between this and the former reading is the angle required, D.

Remove the prism, displace the collimator, and readjust the telescope to view the slit.

Read the vernier.

Replace the prism and take a second observation for minimum deviation.

Take a mean of the two observations.

It would be well to check the result by reversing the prism, so that the face of incidence is made that of refraction, and measuring the deviation in the opposite direction.

Calculate the refractive index from the known angle of the prism and its minimum deviation by means of the formula

$$\mu = \frac{\sin \frac{D + i}{2}}{\sin \frac{i}{2}}.$$

Example.—Enter results thus:

Moving Telescope.			Moving Prism.			For Minimum Deviation.		
Reading Right.	Reading Left.	i	Right.	Left.	i	Through Col.	Through Prism.	μ
150° 56′	31° 10′	59° 50′	174°10′	54° 25′	59° 52′	91° 0′	41° 45′	
157° 00′	37° 15′	59° 52′	171°25	51° 40′	59° 52′	90° 35′	41° 20′	1.66
Mean........		59° 51′			59° 52′	$D =$	49° 15′	

Blank to be filled in by student.

Moving Telescope.			Moving Prism.			For Minimum Deviation.		
Reading Right.	Reading Left.	i	Right.	Left.	i	Through Col.	Through Prism.	μ
Mean........						$D =$		

31. TO DETERMINE THE REFRACTIVE INDEX OF A LIQUID BY MEANS OF A MICROSCOPE.

References.—Watson, p. 495.

Apparatus Required.—A microscope; a beaker with a fine cross or other well-defined object at the bottom; a fine millimeter scale for determining the positions of the microscope tube.

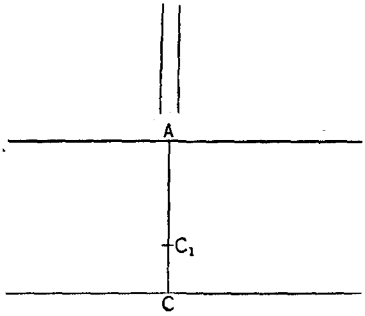

FIG. 22.

Theory of Experiment.—If an object C placed in a vessel partially filled with a liquid (e.g., water) be viewed from a position perpendicularly above the liquid, it will appear at a point C_1 nearer the surface than C, due to the refraction of the liquid.

If A be the point on the surface of the liquid perpendicularly above C, then the refractive index of the liquid is given by equation

$$\mu = \frac{AC}{AC_1}.$$

In order to measure the distances AC and AC_1, a microscope can be used as follows.

Practical Directions.—Scratch on the bottom of a beaker which is at least two inches high a fine cross.

Place the beaker under the object-glass of the microscope, and carefully focus on the cross at the bottom.

Measure with a fine scale, to $\frac{1}{10}$ of a millimeter, the distance between a fixed point on the microscope and a fixed point on the stand.

Denote this distance by δ.

The focussing and measuring should be done three times, and the mean position of the tube taken.

Pour in some liquid and sprinkle some light powder, such as lycopodium, on the surface.

Now focus on the refracted image of the cross, and again measure carefully the distance between the two fixed points, δ_1.

Take a mean of three observations.

Then focus on the lycopodium powder on the surface, taking, as before, a mean of three observations of the distance between the points, δ_2.

The depth AC of the liquid is clearly the difference between the first distance and the last, $\delta - \delta_2$, and the length AC_1 the difference between the second distance and the last, $\delta_1 - \delta_2$.

Deduce these lengths and calculate the value of μ from formula (1).

Example.—Enter results thus:

δ	δ_1	δ_2	AC	AC_1	μ
7.57 7.57 7.58	6.70 6.69 6.69	4.06 4.06 4.05	3.51 3.51 3.53	2.64 2.63 2.64	1.330 1.335 1.337
Mean value of μ..................................					1.334

Blank to be filled in by student.

δ	δ_1	δ_2	AC	AC_1	μ
Mean value of μ..................................					

HEAT.

32. TO CONSTRUCT AND CALIBRATE A SPIRIT THERMOMETER.

References.—Preston, p. 164; Nichols and Franklin, vol. I. p. 153; Ames, p. 207; Knott, p. 214; Barker, p. 272; Hastings and Beach, p. 175; Anthony and Brackett, p. 206; Watson, p. 279.

Apparatus Required.—A glass tube of about ½ mm. bore with a bulb blown on one end; some alcohol; a glass beaker; a small tripod, with iron gauze cover; a bunsen burner; a vessel filled with snow saturated with water; a suitable funnel for filling bulb.

Theory of Experiment.—If the glass bulb be filled with alcohol or other liquid, and heated, the liquid will expand and rise in the tube connected to the bulb. The expansion will be proportional to the increase of temperature, or

$$V = V_0(1 + \alpha t),$$

where V_0 is the volume at zero temperature, and V that at temperature t. The increase of temperature may therefore be measured by measuring the rise of the liquid in the tube. The tube can therefore (assuming the uniformity of the bore) be calibrated in degrees of temperature by determining the position of the liquid for two different temperatures and dividing it proportionally.

Practical Directions.—Blow a suitable bulb on the end of the tube. For $\frac{1}{2}$ mm. bore the bulb should be about 1 cm. diameter, the tube being about $\frac{1}{8}$ of a meter in length.

Connect a small glass funnel to the end of the tube to be filled, by means of a rubber tube fitting each tightly.

Partially fill the funnel with alcohol.

Now gently heat the bulb over the gas-flame thus expelling the air from the bulb.

On cooling the bulb it will be found partially filled with the liquid.

Now gently boil the liquid till it is expelled, and again cool.

The bulb will now be found to be full of liquid, with probably the exception of a small air-bubble in the top or in the stem. To get rid of this final bubble, hold the tube in the hand with the bulb downward, and swing it with a circular motion. The air, being lighter, will be displaced by the liquid, due to its greater centrifugal force.

By continuing this action the air-bubble can be made to rise in the tube, and can finally be expelled by slightly warming the bulb with the hand or in warm water.

Seal the tube by means of a blowpipe flame.

To accomplish this easily the tube should be drawn out at the point for sealing before it is filled.

Warm the liquid till the tube is just full to the sealing point.

The sealing should be done quickly, and the bulb cooled at once to permit the liquid to contract.

If alcohol be used, it should be at about 75° when sealed, so that a good range of temperature can be obtained.

This can be accomplished by keeping the bulb in the glass beaker filled with water heated to about 75° C.

As the boiling-point of alcohol is about 79° C., the thermometer must not be heated to that point.

Now fasten the bulb and tube to a narrow strip of section-paper. Determine the zero-point by putting the bulb in snow saturated with water.

Determine a point at say 60° or 70° by heating it in water.

To do this a mercury thermometer must be used, and simultaneous readings taken.

Divide the section-paper into degrees of temperature.

This experiment requires considerable skill, and the student must not get discouraged if he fails on first trial.

Return the thermometer with your name written on it.

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33. TO TEST THE FIXED POINTS OF A THERMOMETER, AND TO DETERMINE THE STEM-EXPOSURE CORRECTION FOR ANY TEMPERATURE.

References.—Preston, p. 105; Watson, p. 210; Barker, p. 273; Anthony and Brackett, p. 189; Hastings and Beach, p. 165; Ames, p. 202; Knott, p. 195; Nichols and Franklin, vol. I. p. 151.

Apparatus Required.—A thermometer to be tested; a telescope for accurately reading the thermometer; a glass beaker filled with snow saturated with water; a hypsometer and suitable burner.

Theory of Experiment.—(1) By testing the fixed points of a thermometer is meant the determination of the indications of the thermometer corresponding to the freezing-point of water and to the boiling-point of water under 760 *mm.* pressure. Suppose on reading the freezing-point it is found not to be the indicated zero, but to differ from it by a small value

$\pm a$, a being considered plus when the reading is above the zero, and minus when below.

If when the boiling-point is observed the barometric reading differ from 760 mm. by b_1, then the true temperature of the steam is $100 \pm (b_1 \times .037)$, according as the barometric reading is greater or less than 760 mm.

Suppose the reading on the thermometer to differ from this true value by a small quantity $\pm b$, b being $+$ or $-$ according as the thermometer reading is above or below the true reading. Then the total error in the length of the stem for 100 degrees of temperature is $\pm a \pm b$.

Hence a true degree on the thermometer, supposing the tube to be uniform in bore, is indicated by $\frac{100 \pm a \pm b}{100}$ divisions of the thermometer, and therefore any temperature t would be indicated by $\frac{100 \pm a \pm b}{100} \times t$ thermometer divisions from the true zero, or from the zero of the thermometer by

$$\frac{100 \pm a \pm b}{100} \times t \pm a. \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

(2) If when the boiling-point is observed the thermometer be wholly immersed in the hypsometer and the reading taken, and the observation repeated with 30 or 40 degrees of the stem exposed, *it will be found that the readings slightly differ*, owing to the exposure of the stem to the air. Denoting the length of stem exposed by δ, and the difference in the readings by k, then the stem correction per degree is $\frac{k}{\delta}$.

This stem correction will depend not only on the stem exposed, but also on the temperature being determined; and

will be positive or negative according as that temperature is above or below the temperature of the room.

The reading of the thermometer corresponding to any temperature t is therefore

$$\frac{100 \pm a \pm b}{100} \times t \pm a \pm \frac{k\delta_1}{\delta}, \quad . \quad . \quad . \quad (2)$$

or

$$t = \frac{(\text{Ther. reading} \mp a \mp \frac{k\delta_1}{\delta})\,100}{100 \pm a \pm b}, \quad . \quad . \quad . \quad (3)$$

$\frac{k}{\delta}$ being the stem correction for temperature t, and δ_1 the length of stem exposed when the temperature t is taken.

Practical Directions.—Insert the thermometer in the mixture of snow and water, leaving only sufficient of the mercury column exposed to enable you to take the reading.

Read by means of the telescope the indication of the thermometer to $\frac{1}{100}$ of a degree.

This reading gives you the value a.

By means of a split cork insert the thermometer in the hypsometer, and let the steam flow freely for a couple of minutes.

As before, have only sufficient of the stem exposed to enable you to take the reading.

Read the indication again as above.

Read the barometer, and calculate the true temperature of steam, or find it from a chart in the laboratory.

The difference between this and the thermometer reading gives the value b.

Now expose the stem 40 or 50 degrees and read the thermometer again.

Calculate the stem correction per degree of stem exposure.

Calculate the temperature corresponding to a reading of 40 degrees on the thermometer, supposing you can neglect the stem correction.

Find the temperature by the thermometer of the solution provided, and calculate the true temperature.

Example.—Enter results thus:

a	Barometer Reading.	Calculated Temperature of Steam.	Observed Temperature of Steam.	*b*
+ .5	76.3	100.11	100.70	+ .59
Ther. Reading, no stem exposed.	Ther. Reading. 20° stem exposure.	$\frac{k}{\delta}$	Temperature of Solution by Thermometer.	Corrected Temperature.
100.70	100.50	.01	42.30	42.76

Blank to be filled in by student.

a	Barometer Reading.	Calculated Temperature of Steam.	Observed Temperature of Steam.	*b*
Ther. Reading, no stem exposed.	Ther. Reading. 20° stem exposure.	$\frac{k}{\delta}$	Temperature of Solution by Thermometer.	Corrected Temperature.

34. TO DETERMINE THE COEFFICIENT OF EXPANSION OF A LIQUID BY A WEIGHT THERMOMETER.

References.—Carhart, pt. II. p. 30; Preston, p. 173; Knott, pt. I. p. 214; Ames, p. 207; Nichols and Franklin, vol. I. p. 153; Hastings and Beach, p. 169; Anthony and Brackett, p. 208; Barker, p. 291; Watson, p. 219.

Apparatus Required.—A weight thermometer; a hypsometer with suitable stand for use with bunsen burner; a bunsen burner; a beaker; a thermometer; a small clip for holding weight thermometer.

Theory of Experiment.—If a glass tube be filled with glycerine or other liquid at a temperature t, and then heated to another temperature t_1, the liquid will expand and part of it will be expelled from the tube.

Let V_0 denote volume of the tube at the temperature t;
V_1 that at the temperature t_1;
V_2 the total volume of the expanded glycerine at temperature t_1;
δ_0 the density of glycerine at t;
δ_1 the density at t_1;
α the coefficient of the expansion of the glass;
β the coefficient of the expansion of the glycerine;
M_0 the mass of liquid in the tube at t;
M_1 the mass in the tube at temperature t_1.

Then the following relations hold:

$$V_0\delta_0 = M_0; \quad \text{(1)}$$
$$V_1\delta_1 = M_1; \quad \text{(2)}$$
$$V_0\delta_0 = V_2\delta_1; \quad \text{(3)}$$
$$V_1 = V_0\{1 + \alpha(t_1 - t)\}. \quad \text{(4)}$$

Hence, combining (1), (2), and (4),

$$\frac{\delta_0}{\delta_1} = \frac{M_0}{M_1} \times \frac{V_1}{V_0} = \frac{M_0}{M_1}\{1 + \alpha(t_1 - t)\}. \quad \text{(5)}$$

Also $$\frac{\delta_0}{\delta_1} = \frac{V_2}{V_0} \text{ from (3)},$$

and therefore $$\frac{V_2}{V_0} = \frac{M_0}{M_1}\{1 + \alpha\,(t_1 - t)\}.$$

Hence $$\frac{V_2 - V_0}{V_0(t_1 - t)} = \frac{M_0 - M_1}{M_1(t_1 - t)} + \frac{M_0}{M_1}\alpha. \quad . \quad . \quad . \quad (6)$$

But $$\frac{V_2 - V_0}{V_0(t_1 - t)} = \beta,$$

the coefficient of expansion per unit volume per degree of temperature. Hence

$$\beta = \frac{M_0 - M_1}{M_1(t_1 - t)} + \frac{M_0}{M_1}\alpha, \quad . \quad . \quad . \quad . \quad (7)$$

from which β can be calculated if α be known and the other values observed.

Practical Directions.—A suitable weight thermometer can be made from a piece of glass tube 1 cm. diameter, drawn out as in Fig. 23. The bulb AB should be about 7 cm long.

Weigh it carefully to .001 gm., denoting the weight by W.

Fasten the thermometer in the clip for the purpose, and adjust the vessel containing glycerine till the end of the fine tube of the weight thermometer is immersed in the glycerine.

Now, by means of a bunsen burner, slowly heat the glass bulb, thus expelling some of the air by expansion.

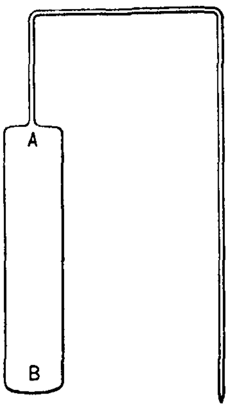

Fig. 23.

Let the bulb cool, and on cooling the glycerine will rise in the tube and partially fill the bulb.

Again slowly heat the bulb until the glycerine begins to boil and again cool, repeating the operation until the last bubble of air is expelled.

Keeping the end of the tube still under the glycerine, cool the bulb to about 1° above the temperature of the room. This can be done by putting the bulb in a beaker of water slightly warmer than the room temperature.

The glycerine must be left for some minutes in the water to secure uniform temperature, the water being slowly stirred with the thermometer all the time.

Read the temperature of the water, t.

Now take the weight thermometer out of the water and carefully dry with a cloth, being careful not to heat it with the hand or the glycerine will expand and some of it drop from the fine tube. On taking it out of the water into the cooler atmosphere of the room it will slightly contract, thus making it possible to weigh it without loss.

Weigh carefully the now filled bulb again to .001 gm.

Denote the weight by W_1.

$$W_1 - W = M_0.$$

Now suspend the weight thermometer inside a hypsometer.

This can easily be done, if the bulb has been properly made, by having a split cork for the top of the hypsometer.

Allow the steam from the hypsometer to flow freely around the bulb.

If it be not convenient to use a hypsometer, the bulb can be suspended in boiling water, and the temperature of the water taken with a thermometer.

If the hypsometer be used, read the barometer and take the temperature t_1 from the chart in the room.

The overflow of glycerine should be caught in a beaker.

Leave the bulb in the hypsometer or water until the glycerine ceases to drop from the open end of the tube.

Weigh again.

Denote the weight by W_2.

$$M_1 = W_2 - W;$$

$$\alpha, \text{ for glass } = .000026.$$

Substitute these results in the formula, and calculate β.

Example.—Enter results thus:

W.	W_1.	M_0.	t.	W_2.	M_1.	t_1.	β.
10.570	17.375	6.805	21.5	17.134	6.564	99.7	.000495

Blank to be filled in by student.

W.	W_1.	M_0.	t.	W_2.	M_1.	t_1.	β.

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35. TO DETERMINE THE COEFFICIENT OF LINEAR EXPANSION OF BRASS.

References.—Watson, p. 214; Preston, p. 98; Hastings and Beach, p. 172; Carhart, pt. II. p. 28; Barker, p. 289; Anthony and Brackett, p. 208; Nichols and Franklin, vol. I. p. 153; Ames, p. 204; Knott, p. 217.

Apparatus Required.—Two microscopes; two brass tubes, one considerably larger than the other; a hypsometer, with rubber tubing to make connections; a beam-compass; a centimeter scale; two thermometers.

Theory of Experiment.—If a brass rod of length l and uniform temperature t be heated until it attains a uniform temperature t_1, it will be found on measurement to have increased in length.

Denote the length at temperature t_1 by l_1. The coefficient of linear expansion between t and t_1 is given by the equation

$$\alpha = \frac{l_1 - l}{l(t_1 - t)}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where α is the coefficient of linear expansion.

If l and l_1 be measured, t and t_1 observed, α can be calculated.

Practical Directions.—By means of corks in the ends, arrange the smaller tube inside the larger one as in Fig. 24. *CD* is the small tube, the coefficient of which is to be deter-

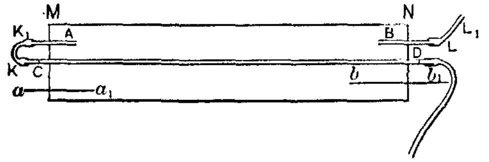

Fig. 24.

mined. *A* and *B* are small glass tubes; aa_1, bb_1, thermometers. KK_1 is a rubber tube connecting the inside of the inner tube *CD* with the inside of the outer tube *MN*. *DP* is a rubber tube connecting the hypsometer to the inner tube; LL_1, a rubber tube for carrying off the steam as it flows out of the outer tube.

Make two sharp knife-cuts, *C* and *D*, at places in the outside portion of the inner tube, convenient for observations, yet as close as possible to the corks in the large tube *MN*.

Focus one of the microscopes on the cut *C* and adjust until the cross-hair of the microscope, being central, is over the centre of the cut.

Clamp the microscope to the table or slab on which the apparatus is placed.

Similarly adjust the other microscope to the cut *D*.

Read carefully the temperatures of the thermometers inside the tube, and take the mean of the two as t.

Light the burner under the hypsometer and let the steam flow freely through the inner tube, outer tube, and again to the air.

Let the steam flow freely for a few minutes till the temperature becomes steady.

Read the barometer, and the temperature of the steam corresponding to the barometric pressure from the chart in the laboratory.

On looking through the microscopes, it will be found that the cuts on the tube have now moved away, one to the right and one to the left of the cross-hairs.

Count the number of micrometer divisions, in each case, between the cross-hairs and the new positions of the cuts.

This may be done by counting the scale divisions in the microscope, or by counting the number of turns of the micrometer head, in each case, required to move the cross-hairs from their original positions to the new positions of the cuts.

The sum of the two, expressed in centimeters, gives the expansion of the rod.

Now measure, by means of a fine scale, the value of each micrometer division.

This can be done by focussing the microscope on the fine scale, the divisions of which are known, and counting the micrometer divisions corresponding to a scale division.

The expansion $l_1 - l$ is thus determined in centimeters.

Now measure, by means of the beam-compass and a centimeter scale, the length l to the nearest millimeter.

Calculate α from formula (1).

Example.—Enter results thus:

t	t_1	$t_1 - t$	Micrometer Divisions in Right Microscope.	Micrometer Divisions in Left Microscope.	$l_1 - l$	l	a .0000
19.5	99.5	80.0	6.5	6.2	.147	100	184

Blank to be filled in by student.

t	t_1	$t_1 - t$	Micrometer Divisions in Right Microscope.	Micrometer Divisions in Left Microscope.	$l_1 - l$	l	a

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36. TO DETERMINE THE COEFFICIENT OF INCREASE OF PRESSURE OF AIR BY MEANS OF A CONSTANT-VOLUME AIR-THERMOMETER.

References.—Nichols and Franklin, vol. I. p. 146; Hastings and Beach, pp. 164 and 182; Carhart, pt. I. p. 39; Anthony and Brackett, pp. 191 and 222; Preston, pp. 129 and 187; Knott, p. 211; Ames, p. 212; Barker, p. 295; Watson, p. 229.

Apparatus Required.—A constant-volume air-thermometer; a metal vessel for snow and water mixture; a hypsometer; a bunsen burner; a telescope.

Theory of Experiment.—Let V_0 be the volume of a mass of gas, M, at 0°C. or T_0 of the absolute scale, and under a

pressure P_0; V, $T_0 + t$, and P the corresponding values when volume, pressure, and temperature change. The law connecting the two sets of values for the same mass is given by the formula

$$\frac{P_0 V_0}{T_0} = \frac{P V}{T_0 + t} = MK. \quad . \quad . \quad . \quad . \quad (1)$$

If the volume be kept constant,

$$\frac{P_0}{T_0} = \frac{P}{T_0 + t}, \quad \text{or} \quad T_0 = \frac{P_0 t}{P - P_0}.$$

If the pressure be kept constant,

$$\frac{V_0}{T_0} = \frac{V}{T_0 + t}, \quad \text{or} \quad T_0 = \frac{V_0 t}{V - V_0}.$$

If α be the coefficient of increase of pressure at constant volume,

$$\alpha = \frac{P - P_0}{P_0 t} = \frac{1}{T_0} = \frac{V - V_0}{V_0 t}. \quad . \quad . \quad . \quad (2)$$

Hence the coefficient of increase of pressure at constant volume is equal to the coefficient of increase of volume at constant pressure.

The formula for the present experiment is

$$\alpha = \frac{P - P_0}{P_0 t}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In the actual working of the experiment there are two corrections which must be applied, which introduce additional terms in the formula. These will be discussed under next section.

Practical Directions.—We shall assume that an air-thermometer of the Jolly type is used.

Observations for Freezing-point.—The bulb of the thermometer, having been filled with dry air, should be first carefully packed in snow or ice saturated with water.

When the temperature becomes steady, raise the adjustable tube of the manometer until the mercury in the stationary one just touches the black glass point in the outer bulb.

Read the level of the mercury in each tube by means of the telescope referred to the graduated scale attached to the instrument. Denote the readings by A and S, and the difference of level by p_0.

Repeat the observations several times and average the result.

Read the barometer, denoting the reading by H_0.

Observe the temperature of the barometer, and also that of the air near the air-thermometer.

If the readings be nearly the same, the mercury columns need not be corrected for temperature.

Observations for Boiling-point.—Insert the bulb of the thermometer in the hypsometer, and boil the water by means of the bunsen flame.

Adjust the manometer as before.

Read again the level of the mercury in each tube, denoting the difference by p_1, the readings by A_1 and S_1.

Repeat the observations as before.

Read the barometer, H_1.

The barometer reading in this case will not usually differ much from H_0 in the preceding case.

The temperature of the steam, t, for the pressure H_1 may be found from a curve in the laboratory.

Corrections.—In making calculations from these observations, two corrections, as mentioned before, must be noted.

(1) *Correction for Expansion of the Glass Bulb.*—The volume of air is not the same in each case on account of the

expansion of the glass bulb. The relation between the two volumes is given by the equation

$$V = V_0(1 + gt), \quad \text{when} \quad g = .000026.$$

Since the difference of temperature is nearly 100,

$$V = V_0(1.0026). \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

(2) *Correction for Stem Exposure.*—The air in the small tube or stem leading from the bulb containing the air to the tube containing the mercury remains approximately at the temperature of the room.

Denoting the volume of this small tube by v, the mass of the air it contains by m, that in the bulb by M_1, we have the relations

$$\frac{PV}{T_0 + t} = M_1K, \qquad \frac{Pv}{T_a} = mk,$$

where T_a is the absolute temperature of the air in the room, and T_0 the absolute temperature of zero centigrade.

Since $\qquad M_1 + m = M = \text{constant},$

we therefore have

$$\frac{P_0V_0}{T_0} + \frac{P_0v}{T_a} = \frac{PV}{T_0 + t} + \frac{Pv}{T_a}. \quad . \quad . \quad . \qquad (2)$$

The ratio of the volume v to V_0 must be determined if it be not, as in most cases, given with the instrument.

Denote this ratio by r. Substitute V_0r for v, $V_0(1 + gt)$ for V, and divide through by V_0. Equation (2) now becomes

$$\frac{P_0}{T_0} + \frac{rP_0}{T_a} = \frac{P(1 + gt)}{T_0 + t} + \frac{rP}{T_a}.$$

Multiply both sides by $\frac{(T_0+t)T_0}{P_0}$, take $1+gt$ out of every term except the first, and we obtain

$$T_0+t=\frac{(1+gt)T_0P}{P_0}\left\{1+\frac{r(T_0+t)}{T_a(1+gt)}-\frac{rP_0(T_0+t)}{PT_a(1+gt)}\right\}$$

$$=\frac{(1+gt)T_0P}{P_0}\left\{1+\frac{rt}{T_a(1+gt)}+\frac{r[T_0P-(T_0+t)P_0]}{PT_a(1+gt)}\right\}.$$

Assuming $T_0P=(T_0+t)P_0$ in the small term and neglecting gt in the denominator of the second term, which is also small, we have

$$T_0+t=\frac{(1+gt)T_0P}{P_0}\left(1+\frac{rt}{T_a}\right).$$

Hence

$$\alpha=\frac{1}{T_0}=\frac{\left\{P\left(1+gt+\frac{rt}{T_a}\right)-P_0\right\}}{P_0t}\quad . \quad . \qquad (3)$$

Denoting T_0 as 273° in the small term, t_a the temperature centigrade, and substituting for P_0 and P_1 the values H_0+p_0 and H_1+p_1, the formula becomes

$$\alpha=\frac{(H_1+p_1)\left\{1+gt+\frac{rt}{273+t_a}-(H_0+p_0)\right\}}{(H_0+p_0)t}\qquad (4)$$

In the Jolly pattern air-thermometer used in this laboratory $r=\frac{2.35}{124.25}=.0190$, the volume of the stem per centimeter being .0227. In the Groves pattern $r=.0150$.

Precautions.—(1) The tube supporting the bulb is delicate and easily broken; it must therefore be carefully handled, especially when packing in snow.

(2) Before taking the bulb out of the hypsometer or turning off the gas-flame, lower the adjustable tube of the

manometer, otherwise the mercury may be forced into the bulb as it cools.

(3) In the Jolly pattern air-thermometer do not touch the three-way tap on the left-hand side. If this be turned either way, mercury will be spilled.

(4) Be sure the hypsometer contains water before heating.

Example.—Enter results thus:

FREEZING-POINT OBSERVATIONS.

H_0	Temperature of Bar.	Manometer.		p_0	t_a
		S	A		
76.66	16.2	50.21	50.53	0.32	16.1
		50.21	50.52	.32	16.1
		50.20	50.52	.32	16.0
	16.0	50.19	50.51	.32	15.8
76.66	16.1	50.20	50.52	.32	16.0

BOILING-POINT OBSERVATIONS.

H_t	Temperature of Bar.	Manometer.		p_1	t_a
		S_1	A_1		
76.70	16.2	50.21	77.63	27.42	16.3
		50.20	77.63	27.43	16.4
		50.22	77.63	27.41	16.4
76.68	16.2	50.20	77.62	27.42	16.4
76.69	16.2	50.21	77.63	27.42	16.4

$P_0 = 76.66 + 0.32 = 76.98;$

$P_1 = 76.69 + 27.42 = 104.11;$

$t =$ temperature of steam under pressure 76.69 $= 100.25;$

$$\alpha = \frac{104.11 \left\{1.0026 + \frac{1.91}{273 + 16.4}\right\} - 76.98}{76.98 \times 100.25} = .003650.$$

Blanks to be filled in by student.

FREEZING-POINT OBSERVATIONS.

H_0	Temp. Bar.	S	A	p_0	t_a

BOILING-POINT OBSERVATIONS.

H_1	Temp. Bar.	S_1	A_1	p_1	t_a

$P_0 =$

$P_1 =$

$t =$

$\alpha =$ $\quad =$

37. TO DETERMINE THE COEFFICIENT OF INCREASE OF VOLUME OF AIR BY MEANS OF A CONSTANT-PRESSURE AIR-THERMOMETER.

References.—As in preceding experiment.

Apparatus Required.—A suitable glass bulb; a hypsometer; a bunsen burner; a glass vessel of not less than 8 cm. diameter and 25 cm. deep; a thermometer.

Theory of Experiment.—It has been shown in the experiment on the constant-volume air-thermometer that the coefficient of increase of pressure at constant volume is equal to the coefficient of increase of volume at constant pressure, or

$$\alpha = \frac{P - P_0}{P_0 t} = \frac{V - V_0}{V_0 t} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In the present experiment it is proposed to keep the pressure constant, and to measure the coefficient by means of increase of volume from the equation

$$\alpha = \frac{V - V_0}{V_0 t}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

V_0 and V being the volumes at 0°C. and t respectively.

If the volumes be taken at temperatures t_1 and t_2, then

$$\alpha = \frac{V_1 - V_2}{V_2(t_1 - t_2)}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where α is the coefficient of increase of volume per degree of temperature between t_1 and t_2.

If a glass bulb of weight W filled with air at temper-

ature t_1 be immersed in water at a temperature t_2 (t_2 being lower than t_1), so that no air is permitted to escape, it will become partially filled with water, due to the contraction of the air in the bulb.

Denote the weight of the partially filled tube by W_1.

If the tube be now filled with water and again weighed, its weight being denoted by W_2, the volume of the bulb at temperature t_2 is obviously

$$W_2 - W,$$

while the volume when filled with air at temperature t_1 is given by the equation

$$V_1 = (W_2 - W)\{1 + .000026(t_1 - t_2)\}, \quad . \quad (1)$$

.000026 being the coefficient of expansion of glass.

The volume of air V_2 in the partially filled tube is obviously

$$W_2 - W - (W_1 - W),$$

or

$$V_2 = W_2 - W_1. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence

$$\alpha = \frac{V_1 - V_2}{V_2(t_1 - t_2)}$$

$$= \frac{(W_2 - W)\{1 + .000026(t_1 - t_2)\} - (W_2 - W_1)}{(W_2 - W_1)(t_1 - t_2)}$$

$$= \frac{W_1 - W + (W_2 - W)\{.000026(t_1 - t_2)\}}{(W_2 - W_1)(t_1 - t_2)}. \quad . \quad (3)$$

If W, W_1, W_2, be obtained, t_1 and t_2 observed, the value of α can be calculated.

Practical Directions.—A suitable bulb for the experiment can be made from a piece of glass tube drawn out at each end, as in Fig. 25.

FIG. 25.

Make the part AB about 10 cm. in length from a piece of tube 2 cm. in diameter.

The bulb should be drawn out to a very fine point at each end.

Weigh the bulb, denoting the weight by W.

Seal the tube at one end, and by means of a split cork insert it, sealed end down, into the hypsometer, with about 1 cm. of the open end protruding from the cork.

Let the steam flow freely for about ten minutes.

Seal the open end by means of a blowpipe or bunsen burner. Read the barometer, and find the corresponding temperature of steam, t_1.

Now fill the glass vessel, mentioned under apparatus, nearly full of water at about the temperature of the room. Holding the end of the bulb under water, break off a small bit of the top of the tube, and immediately the tube will become partially filled with water, due to the cooling of the air and the consequent change of pressure in the bulb. The bulb should be kept vertical, open end down, to prevent the escape of the air.

The pressure in the bulb is made up of two parts, the pressure of the air in the bulb, and the pressure due to the presence of aqueous vapor.

This pressure is equal to the barometric pressure plus the difference in the head of the water in the bulb and vessel, or,

$$\text{barometric pressure} + \text{pressure of water} =$$
$$\text{pressure of air} + \text{aqueous vapor pressure.}$$

Hence we can correct for aqueous vapor pressure by making the pressure due to difference of head of water exactly equal to it, thus making the pressure due to the air in the bulb exactly equal to the barometric pressure, as was the case when the bulb was in the hypsometer.

Calculate, therefore, the depth of water equal to the aqueous vapor pressure at the temperature of the water, and depress the bulb until a difference between the surface of the water in the vessel and bulb equal to it is obtained.

In order to do this, read from the chart in the laboratory the pressure of the aqueous vapor at temperature of water. Denoting this by P, we have

$$\frac{h}{13.596} = P, \quad \text{or} \quad h = P \times 13.596,$$

where h is the difference of head required, and 13.596 the specific gravity of mercury, the vapor pressure and barometric pressure being expressed in centimeters of mercury. While holding the bulb in the water at depth h, seal the open end with wax.

A small piece of suitable wax can be kept attached to the bottom of the vessel inside, and the depth of the water regulated so as to give h just as the open end of the tube touches the bottom of the vessel.

It will be found convenient to have a piece of stiff wire, to use as a handle, twisted round the bulb.

Stir the water in the vessel, and read the temperature t_1.

Remove the bulb, being careful not to lose any of the water out of it.

Dry and weigh. Denote the weight by W_1.

Now fill the bulb with water.

This can be done by the method employed in filling the weight thermometer.

Weigh the bulb when full of water, denoting the weight by W_2.

Substitute these weights in the formula, and calculate α.

Example.—Enter results thus:

W	t_1	t_2	W_1	W_2	α
9.750	100.25	19.50	14.463	30.268	.00370

Blank to be filled in by student.

W	t_1	t_2	W_1	W_2	α

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38. TO DETERMINE THE SPECIFIC HEAT OF COPPER—METHOD OF MIXTURES.

References.—Preston, pp. 211 and 215; Carhart, pt. II. p. 44; Barker, p. 283; Anthony and Brackett, p. 193; Watson, p. 288; Knott, p. 199; Ames, p. 217; Nichols and Franklin, vol. I. p. 164; Hastings and Beach, p. 188.

Apparatus Required.—A regulation cylindrical heater, with hypsometer attachments and calorimeter; two thermometers.

Theory of Experiment.—By the *specific heat* of a substance is meant the ratio of the quantity of heat required to

raise the temperature of a mass of the substance one degree to the quantity necessary to raise an equal mass of water one degree. By a *unit of heat* is meant the quantity of heat necessary to raise one gram of water through one degree.

If a known mass of copper, m, be heated to a temperature t, and then suddenly plunged into a known mass of water, M, at a lower temperature, t_2, and the water stirred until the water and copper have a uniform temperature, t_1, the heat lost by the mass m of copper is equal to $cm(t - t_1)$ units, where c is the specific heat of copper. The heat absorbed by the water is $M(t_1 - t_2)$. Hence

$$cm(t - t_1) = M(t_1 - t_2),$$

or

$$c = \frac{M(t_1 - t_2)}{m(t - t_1)}. \quad . \quad . \quad . \quad . \quad (1)$$

Formula (1), however, takes no account of the heat taken up by the vessel containing the water.

Suppose the vessel to be copper, of mass m_1, and to become uniformly heated with the water.

Then as before the heat lost by the copper mass m is $cm(t - t_1)$, that gained by the water would be $M(t_1 - t_2)$, that gained by the calorimeter would be $cm_1(t_1 - t_2)$, since the specific heat of the calorimeter is the same as that of the heated mass m. Hence

$$cm(t - t_1) = M(t_1 - t_2) + cm_1(t_1 - t_2).$$

Therefore

$$c = \frac{M(t_1 - t_2)}{m(t - t_1) - m_1(t_1 - t_2)}, \quad . \quad . \quad (2)$$

a formula from which c can be calculated if observations be made for the other terms.

Having determined the specific heat of copper, the calo-

rimeter can now be used to determine the specific heat of any other substance.

Thus if c be the specific heat of copper, c_1 that of another substance of mass m to be determined, all the other conditions remaining the same,

$$c_1 m(t - t_1) = M(t_1 - t_2) + cm_1(t_1 - t_2),$$

or

$$c_1 = \frac{(t_1 - t_2)(M + cm_1)}{m(t - t_1)}, \quad . \quad . \quad . \quad . \quad (3)$$

in which c is known.

The value cm_1 is called the "water equivalent" of the calorimeter.

Practical Directions.—Weigh carefully m, the mass of copper the specific heat of which is to be determined.

A suitable mass of copper can be made by twisting bare copper wire around a lead-pencil, making a mass about two inches in length and one inch in diameter. The hole in the centre will be a suitable place in which to insert the thermometer.

Through a cork in the top suspend, by a thread, this mass inside the cylindrical heater.

Adjust the length of the thread till the mass is about halfway down the heater.

Let a thermometer, through the cork, down into the centre of the mass.

Turn on the steam from the hypsometer, and let it flow steadily for about half an hour, or until the thermometer shows a steady temperature between 95° and 100°. A temperature of about 98° can usually be obtained.

While the steam is flowing, weigh carefully the calorimeter, which should be of copper, and stirrer, m_1.

Partially fill the calorimeter with water and weigh again, W.

$$M = W - m_1.$$

a quantity of water of weight W_1 and temperature t_1, and the whole stirred until the ice is melted and the water is of uniform temperature t.

The heat absorbed by the ice without changing its temperature is LW, where L is the latent heat of fusion of ice.

The weight W has furthermore been raised to a temperature t, so that the total heat taken up by the ice in melting and raising it from 0° C. to t is

$$LW + Wt.$$

The heat lost by the water is

$$W_1(t_1 - t).$$

Hence $$LW + Wt = (t_1 - t)W_1,$$

and therefore $$L = \frac{W_1(t_1 - t)}{W} - t. \quad . \quad . \quad . \quad . \quad (1)$$

In this case we have neglected the loss of heat of the calorimeter.

Denoting by C the specific heat of the calorimeter, and its weight by W_2, CW_2 is its water equivalent, so that the heat loss is really

$$(W_1 + CW_2)(t_1 - t).$$

Hence $$LW + Wt = (W_1 + CW_2)(t_1 - t),$$

and therefore $$L = \frac{(W_1 + CW_2)(t_1 - t)}{W} - t, \quad . \quad . \quad (2)$$

from which, if the necessary observations be made, L can be calculated.

Practical Directions.—Weigh carefully the calorimeter and stirrer, W_2.

Partially fill the calorimeter with water warmed until it is about 7° or 8° above the temperature of the room.

Weigh again, denoting the weight by m.

Then, W_1, the weight of water, is equal to $m - W_2$.

Wrap a piece of ice in a dry cloth and break it into small pieces with a mallet.

Wrap pieces of cloth around the points of the crucible-tongs, and pack ice around them to cool them to 0° C.

Stir the water in the calorimeter and read carefully the temperature t_1 before dropping in the ice.

Drop in small pieces of ice with the tongs (carefully drying each piece on the cloth before so doing), and stir the calorimeter steadily.

Continue the process until, all the ice in the calorimeter being melted, the temperature of the water is as much below the temperature of the room as it was above before beginning to put in the ice.

Read the temperature t_2.

Weigh again the calorimeter, denoting the weight by m_1.

Then W, the weight of ice added, is equal to $m_1 - m$.

C, the specific heat of the substance of which the calorimeter is made, is supposed known, and hence CW_2 is known.

Calculate L from formula (2).

Example.—Enter results thus:

W_2	C	m	$\frac{W_1}{(m - W_2)}$	t_1	t	m_1	$\frac{W}{(m_1 - m)}$	L
45.2	.095	95.7	50.5	20.2	12	100.6	4.9	79.5

Blank to be filled in by student.

W_2	C	m	$\frac{W_1}{(m - W_2)}$	t_1	t	m_1	$\frac{W}{(m_1 - m)}$	L

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40. TO DETERMINE THE LATENT HEAT OF STEAM.

References.—Preston, p. 304; Nichols and Franklin, vol. I. p. 171; Anthony and Brackett, p. 228; Hastings and Beach, p. 191; Ames, p. 237; Carhart, pt. II. p. 74; Watson, p. 248; Barker, p. 325; Knott, p. 224.

Apparatus Required.—A suitable calorimeter; a boiler with suitable drying apparatus attachment; a thermometer.

Theory of Experiment.—By the *latent heat* of steam is meant the heat required to convert a gram of water at 100° C. into steam without altering its temperature.

Suppose M, a mass of water in a calorimeter, the mass and specific heat of the calorimeter being m_1 and c respectively, to be at a uniform temperature t, and that there is passed into it a mass of dry steam m_2 at a temperature t_1, which on entering the water is condensed, the whole being brought to a uniform temperature t_2; then, denoting the latent

heat of steam by L, the amount of heat given out by the steam is

$$Lm_2 + m_2(t_1 - t_2),$$

and the heat gained by the water and calorimeter is

$$(M + cm_1)(t_2 - t).$$

Hence $$Lm_2 + m_2(t_1 - t_2) = (M + cm_1)(t_2 - t),$$

or $$L = \frac{(M + cm_1)(t_2 - t)}{m_2} - (t_1 - t_2). \quad . \quad . \quad . \quad (1)$$

L can be calculated from (1) if the necessary observations be made.

Practical Directions.—Weigh carefully the stirrer and calorimeter m_1.

Partially fill the calorimeter with water and weigh again, denoting the weight by W. Then

$$M = W - m_1.$$

The temperature of the water should be reduced as nearly to 0° C. as possible, and when heated by the steam should be raised as much above the temperature of the room as it was previously below it.

If the temperature of the water be 5° C., the room being at 17° C., the water can be raised to 29°, giving a rise of 24°.

The water should be stirred just before the steam is allowed to flow into it, and the temperature t read.

A special arrangement for a boiler or hypsometer is necessary to dry the steam, and prevent it condensing and thus losing its latent heat before it reaches the water of the calorimeter.

A suitable arrangement is to let the outflow of steam be from a spiral tube inside the steam-chamber of the hypsometer or boiler, the spiral being so adjusted that while the

steam flows through it, it is also surrounded by the steam, the result being that the steam which passes into the water becomes thoroughly dried.

A short piece of rubber tubing makes a suitable connection on the outside.

The rubber tubing should be, however, carefully jacketed with cotton wool or felt, so as to prevent condensation of the steam before reaching the water.

Let the steam flow freely for a couple of minutes to permit the connection to get thoroughly hot and dry.

Pinch the end of the rubber tubing and insert in into the calorimeter.

Let the steam flow for a few minutes, stirring the water all the time.

When the required temperature is approached, pinch the tube again and quickly remove it from the water.

Stir the water and read the thermometer, t_2, at its highest point.

Now weigh the calorimeter again, W_1.

$$m_2 = W_1 - W.$$

Read the barometer B and obtain t_1 from the temperature curve for steam.

Substitute in the formula and calculate L.

Example.—Enter results thus:

m_1	W	M	t	B	t_1	t_2	W_1	m_2	L
161.11	336.61	175.50	5.9	75.18	99.7	29.0	343.86	7.25	535.5

Blank to be filled in by student.

m_1	W	M	t	B	t_1	t_2	W_1	m_2	L

MAGNETISM.

41. TO OBTAIN A REPRESENTATION OF LINES OF FORCE WITH IRON FILINGS, AND TO BLUE-PRINT THEM.

References.—Watson, pp. 589–607; Hastings and Beach, p. 355; S. Thompson, pp. 105–113; Ames, p. 347; Carhart, pt. II. p. 310; Anthony and Brackett, p. 259; Nichols and Franklin, vol. II. p. 27; Barker, p. 633.

Apparatus Required.—A selection of permanent magnets; iron filings; a sprinkler for filings; glass plates; blue-print paper; some disks of soft iron.

Theory of Experiment.—Iron being a paramagnetic metal, if free to move in the vicinity of a magnetic field, will tend to set itself in the strongest part of the field. If, therefore, a glass plate be laid over a bar magnet and iron filings be sprinkled uniformly over it, the filings will set themselves along the lines of force when the plate is vibrated. After the vibration the filings will be very dense just around the poles, where the field is strongest, and will be thinnest near the corners and sides of the plate, where the field is weakest.

Practical Directions.—Select a glass plate at least 4 inches longer than the magnet and about 8 inches wide. Place the magnet to be investigated centrally under the plate and sprinkle the iron filings in a thin even coating all over the plate.

Hold down the plate with one hand, and vibrate it gently by sharp taps of the fingers of the other.

Lay the plate on a piece of blue-print paper in the sun, and after exposing for five or ten minutes, depending on the sensitiveness of the paper, wash in water.

The following curves should be obtained:

(1) From a simple bar magnet.

(2) From a horseshoe magnet.

(3) From two bar magnets with like poles together.

(4) From two bar magnets with unlike poles together.

(5) From a bar magnet with a disk of soft iron in its field.

(6) From a horseshoe magnet with the keeper an inch from the poles.

(7) From the end of a bar magnet.

To be Noted and Explained in:

(1) The uniform distribution of the lines and concentration of the filings around the poles.

(2) The concentration and straightness of the lines between the poles, and the curvature and thinness of the lines further away.

(3) The position of the two neutral points and the weak nature of the field.

(4) The position of the neutral point, and the concentrated field between the poles.

(5) The crowding of the lines into the soft-iron end of the field.

(6) The same as in (5), and the absence of lines elsewhere.

(7) The radial nature of the field around the pole.

42. TO MAP THE MAGNETIC FIELD ABOUT A MAGNET, AND TO DETERMINE THE MOMENT OF THE MAGNET BY FINDING THE NEUTRAL POINT IN ITS FIELD.

REFERENCES.—Ames, p. 351; Carhart, p. 316; S. Thompson, p. 124; Watson, p. 598; Barker, p. 631; Nichols and Franklin, pp. 21–25; Anthony and Brackett, pp. 259–262; Hastings and Beach, p. 361.

Apparatus Required.—A bar magnet; a small compass-box; a drawing-board; a large sheet of paper; a set-square; a pair of dividers; a centimeter scale.

Theory of Experiment.—If a compass-needle be brought near to a magnet, it will be found to take up a fixed direction under the action of the magnet and the earth's field. This direction is approximately the direction of the line of magnetic force passing through the centre of the compass.

Suppose A and B to be the positions of the ends of the compass-needle.

If now the compass be moved so that the point previously at A is at B, the new direction of the line of force can be marked by marking the new position C of the point previously at B.

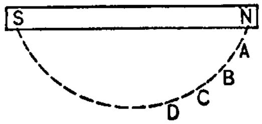

FIG. 26.

The process being continued, the direction of the line can be followed until it goes either off the paper or back to the magnet at another point. By repeating the process a map of the magnetic field can be made.

In mapping the magnetic field, a point will be found where the action of the earth is exactly balanced by the action of the magnet. At this point, the neutral point, the

needle of the compass not being under the control of any directive force, will take any position indifferently. No line of force will therefore pass through this point.

(1) Suppose the magnet be placed in the magnetic meridian with its N pole pointing north, then the neutral point, if the magnet be a simple one, that is, having only two poles, will lie on the perpendicular to the magnet at its middle point.

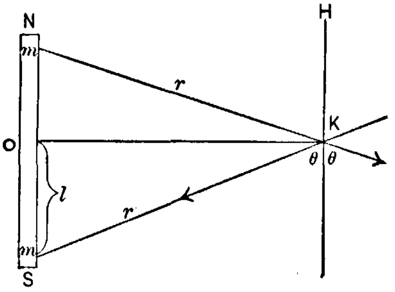

FIG. 27.

Let NS denote the magnet, in the meridian, K the neutral point, OK being perpendicular to NS.

Then the force acting on the needle at K, due to m the strength of each pole, is $\frac{m}{r^2}$, r being the distance of the pole from the needle. The direction is shown by arrows (Fig. 27). Resolving these forces along the meridian, we have

$$H = \frac{2m}{r^2} \cos \theta,$$

since the earth's horizontal component, H, is exactly balanced by the magnet.

Now $$\cos \theta = \frac{l}{r},$$

where l is half the length of the magnet.

Hence $$H = \frac{2ml}{r^3} = \frac{M}{r^3},$$

or $$M = Hr^3, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where M is the moment of the magnet.

(2) Suppose the magnet to be placed with its S pole pointing north.

Then it is evident that since the action of the magnet on a needle north of S is to turn the north pole toward S, while the earth's field tends to turn it in exactly the opposite direction, a neutral point lies directly north of S.

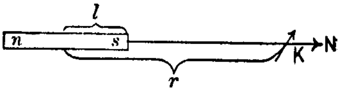

FIG. 28.

Suppose it at a distance r from the centre of the magnet. The attraction of s on the needle at K is $\frac{m}{(r-l)^2}$, while the repulsion of n in the opposite direction is $\frac{m}{(r+l)^2}$.

Hence the total force on the needle due to the magnet is

$$\frac{m}{(r-l)^2} - \frac{m}{(r+l)^2}, \quad \text{or} \quad \frac{4lrm}{(r^2-l^2)^2},$$

which is equal to $$\frac{2Mr}{(r^2-l^2)^2},$$

where M is the moment of the magnet.

Hence $H = \frac{2Mr}{(r^2-l^2)^2}$, since the needle is in equilibrium,

or $$M = \frac{H(r^2-l^2)^2}{2r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

(3) Suppose the magnet to take up a position other than the meridian, as AB (Fig. 29),

Let K be the neutral point.

Then, resolving along the meridian, we have

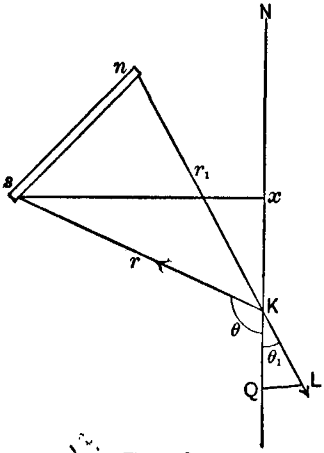

FIG. 30.

$$H = \frac{m}{r_1^2}\cos\theta_1 \pm \frac{m}{r^2}\cos\theta,$$

θ and θ_1 being the angles which the lines drawn from the poles to the needle make with the meridian. The sign between the two terms is — if either θ or θ_1 be greater than 90°.

Hence

$$2lH = M\left\{\frac{\cos\theta_1}{r_1^2} \pm \frac{\cos\theta}{r^2}\right\},$$

or $$M = \frac{2Hlr^2r_1^2}{r^2\cos\theta_1 \pm r_1^2\cos\theta}.$$

Practical Directions.—(1) Fasten a large sheet of paper on a drawing-board.

Lay down on this, by means of a magnetic needle, the direction of the magnetic meridian.

Place the magnet with its edge along this line, its N pole pointing north.

Erect a perpendicular to the magnet from its middle point and move the compass-box along this line until the needle is at the neutral point. When the centre of the needle is at this point the needle will lie in any direction indifferently, assuming of course that the needle is a very short one compared with its distance from the magnet.

To make sure that the neutral point has been found, cause the needle to spin round by means of another magnet

or piece of iron, and note the direction of the needle on coming to rest. When the needle ceases to take up a fixed position the required point has been found.

Mark the position of the compass, and find its centre.

Measure the distance r.

Take for the equivalent length of the magnet $\frac{5}{6}$ the length of the bar.

Find the value of H from the chart of the room.

Substitute this value in formula (1) and calculate M.

(2) Now place the magnet with its S pole pointing north, and move the compass along the meridian until the position of the neutral point is again found.

Measure r as before, and calculate M, formula (2).

(3) Place the magnet in some other position, corresponding to ns in Fig. 29. Plot carefully the magnetic field around the magnet. It will be found that near one point the lines of force bend away, as in Fig. 30. The neutral point lies within this space between the curves.

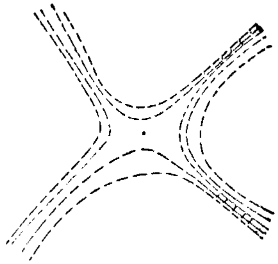

FIG. 30.

Adjust the position of the needle as before till no directive force acts on it.

Measure r and r_1.

Drop perpendiculars, as LQ and sx (Fig. 29), on the direction of the meridian line through K.

$$\text{Then} \quad \cos\theta_1 = \frac{KQ}{LK}; \quad \cos\theta = -\frac{Kx}{r}.$$

Measure the distances corresponding to KQ, LK, Kx.

Substitute in formula (3) and calculate M.

Show diagram in each case.

Example.—Enter results thus:

$H = .1584.$

	r	r_1	l	KQ	Kx	LK	M
1st case	24.5	....	....	....	....	....	2290
2d case	32.0	....	7.5	....	....	....	2310
3d case	30.2	22.6	....	3.95	3.29	8.33	2270
Mean value of M....................................							2291

Blank to be filled in by student.

$H =$

	r	r_1	l	KQ	Kx	LK	M
1st case							
2d case							
3d case							
Mean value of M....................................							

---

43. TO DETERMINE THE MOMENT OF A MAGNET BY OSCILLATION IN A UNIFORM FIELD OF KNOWN INTENSITY.

References.—S. Thompson, p. 121; Watson, p. 604; Ames, p. 352; Nichols and Franklin, vol. II. p. 24; Anthony and Brackett, p. 268; Carhart, pt. II. p. 319; Hastings and Beach, p. 364; Barker, p. 691.

Apparatus Required.—An oscillation-box with suspension; several magnets of different sizes and corresponding torsion weights; a micrometer-gauge; a stop-watch; a compass.

Theory of Experiment.—If a magnet, of moment M, be

allowed to oscillate in a uniform magnetic field H, the law of its vibration is expressed by the formula

$$MH = \pi^2 n^2 K,$$

where n is the number of transits per second, and K the moment of inertia of the magnet. If observation be made for K and n, H being known, M can be calculated.

Practical Directions.—Lay down a meridian line with the compass.

If the bottom of the oscillation-box be provided with a mirror which has a line ruled centrally on it and parallel to the sides of the box, it will be sufficient to set one side of the box along the meridian line. The line on the mirror is to serve as the middle point of the swing of the magnet.

Attach the torsion weight, and after it has come to rest turn the suspension-head until the weight lies along the line on the mirror. Replace the weight by the magnet, being careful to have the N pole pointing north.

Set the magnet swinging through 15° or 20 .

Observe the time, t, of fifty transits past the median line,

$$n = \frac{50}{t}.$$

Measure the length, l, and horizontal thickness, b, of the magnet to $\frac{1}{10}$ of a millimeter.

Weigh the magnet to a centigram.

Calculate the moment of inertia from formula

$$K = W\left(\frac{l^2 + b^2}{12}\right),$$

where W is the weight of the magnet.

Assume the value of H and calculate the moment of the magnet from formula

$$M = \frac{\pi^2 n^2 K}{H}.$$

Make observations for several magnets of different dimensions.

Example.—Enter results thus:

$H = .1489.$

No. of Magnet.	b	l	Weight, Gms.	K	Time of 50 Transits.	Trans. per Second (n).	M
17	8.8	1.2	53 31	350	420″	0.1189	327.8
18	8.8	1.2	53.81	354	440″	0.1136	302.8
19	10.4	1.0	25.18	229	332″	0.1506	341.2
20	14.8	2.0	154.77	2877	500″	0.1000	190.7

Blank to be filled in by student.

$H =$

No. of Magnet.	b	l	Weight, Gms.	K	Time of 50 Transits.	Trans. per Second (n).	M

44. TO COMPARE THE MOMENTS OF TWO MAGNETS BY OSCILLATION.

References.—As in previous experiment.

Apparatus Required.—Two bar magnets; a stirrup bored to fit the magnets and provided with clamps for fixing them rigidly together; a bell-jar or box with suspension; a torsion-weight; a stop-watch; a compass.

Theory of Experiment.—If two magnets, which are rigidly connected together, so that they may be suspended parallel and in the same vertical plane, be vibrated under the control of a constant magnetic force, the ratio of their moments can be readily obtained. For, if they be vibrated (1) with their like poles together, (2) with their like poles opposite, and the number of transits per second, n_1 and n_2 respectively, be noted, we have

$$M_1 = m_1 + m_2, \quad \ldots \ldots \quad (1)$$

$$M_2 = m_1 - m_2, \quad \ldots \ldots \quad (2)$$

where M_1 and M_2 are the respective moments of the systems in the two cases, and m_1 and m_2 the moments of the separate magnets. We also have

$$M_1H = (m_1 + m_2)H = \pi^2 n_1^2 K, \quad \ldots \quad (3)$$

$$M_2H = (m_1 - m_2)H = \pi^2 n_2^2 K, \quad \ldots \quad (4)$$

H being the constant controlling force, which is in this case the earth's horizontal component.

By combining the above we have

$$\frac{m_1}{m_2} = \frac{n_2^2 + n_1^2}{n_1^2 - n_2^2}. \quad \ldots \ldots \quad (5)$$

This gives a very convenient method of comparison, and is practically independent of the size or shape of the magnets.

Practical Directions.—Having laid down a meridian line, hook in the torsion weight and let it come to rest.

Turn the suspension-head around so that when the magnets are suspended they will lie along the meridian.

Clamp the magnets in the stirrup so that they are suspended near the middle of their lengths and with their like poles in the same direction.

Lift off the torsion-weight and hook on the magnets, being careful not to have the suspension fly around in doing so, and that the N poles of the magnets are towards the north.

Let the system come to rest, and mark the point in the bell-jar at each end of the magnets.

Set them swinging by means of another magnet.

Note the time of fifty transits past the marked point.

Reverse the lower magnet, being careful to clamp it in the middle as before.

Observe again the time of fifty transits past the same point.

Obtain the number of transits, n_1 and n_2, in each case, and calculate the ratio m_1 and m_2 from formula (5).

Example.—Enter results thus:

Time of 50 Transits with Like Poles together.	Time of 50 Transits with Like Poles opposite.	n_1	n_2
455	1140	0.1099	0.0438

$$\frac{m_1}{m_2} = \frac{(0.1099)^2 + (0.0438)^2}{(0.1099)^2 - (0.0438)^2} = 1.379.$$

Blank to be filled in by student.

Time of 50 Transits with Like Poles together.	Time of 50 Transits with Like Poles opposite.	n_1	n_2

$$\frac{m_1}{m_2} = \qquad\qquad =$$

45. TO FIND THE MOMENT OF A MAGNET BY THE DEFLECTION METHOD.

References.—Watson, p. 600; Ames, p. 353; Nichols and Franklin, vol. II. p. 23; Anthony and Brackett, p. 268; S. Thompson, p. 124; Carhart, pt. II. p. 318; Hastings and Beach, p. 361; Barker, p. 691.

Apparatus Required.—A magnetometer; a magnet whose moment is to be determined.

Theory of Experiment.—Let a magnet of length $2l$ be placed so that the line of its axis is at right angles to the magnetic meridian, and in line with the centre of a magnetic needle. Let the distance between the centre of the magnet and the needle be denoted by d, Fig. 31.

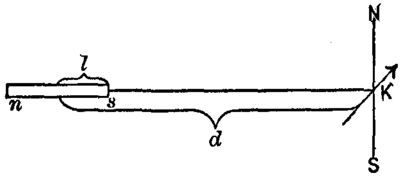

FIG. 31.

The attraction due to the pole n on a pole K will be

$$\frac{mm_1}{(d-l)^2},$$

and the repulsion due to s will be

$$\frac{mm_1}{(d+l)^2},$$

where m is the strength of the poles of the magnet, and m_1 that of the needle at K.

The total attraction therefore will be

$$\frac{mm_1}{(d-l)^2} - \frac{mm_1}{(d+l)^2} \text{ or } \frac{4dlmm_1}{(d^2-l^2)^2}.$$

If the needle be deflected through an angle θ, the moment of the couple acting on the needle will be

$$\frac{4dlmm_1 \cos\theta}{(d^2-l^2)^2},$$

the length of the needle being considered negligible.

Since the needle is in equilibrium, this moment must be equal to that of the earth's magnetic couple.

Hence $$\frac{4dlmm_1 \cos\theta}{(d^2-l^2)^2} = Hm_1 \sin\theta, \quad . \quad . \quad . \quad . \quad (1)$$

H being the earth's horizontal component.

Denoting the moment of the magnet by M, substituting this for $2ml$, and solving, we get

$$M = H\frac{(d^2-l^2)^2}{2d} \tan\theta. \quad . \quad . \quad . \quad . \quad (2)$$

By means of this formula M can be calculated if H be known, d, l, and θ observed.

Practical Directions.—A suitable magnetometer for this experiment may be made by pivoting a light magnetic needle, provided with long pointers, at the zero of a scale graduated toward both ends, and having at fixed distances from the scale, and parallel to it, linear scales for reading the deflections.

Place the magnetometer so that the needle, when in the magnetic meridian, is at right angles to the direction of the magnetometer scale.

If a compass-box, for reading the deflections directly in degrees, be used, the box should be so adjusted that the north and south line is at right angles to the scale.

If a linear scale, as suggested above, be used for reading deflections, the zeros of these scales should be in a line at right angles to the direction of the magnetometer scale.

Place the magnet whose moment is to be determined so that it lies along the magnetometer scale, its N pole pointing to the needle and at a distance from it of from 25 to 50 cm.

If r and r_1 denote respectively the distance of the N and S poles of the magnet from the needle, then

$$d = \frac{r + r_1}{2}.$$

Read the deflections on the linear scales.

Reverse the magnet, the S pole being now a distance r from the needle, and again read deflections.

Similarly place the magnet at a distance r on the opposite side of the needle, reversing as before and reading deflections.

Denote the mean of the eight deflection readings by δ.

Measure the distance between the two linear scales, denoting this length by $2a$; then

$$\tan \theta = \frac{\delta}{a}.$$

To find the length l, measure by means of a centimeter scale the length of the magnet, and take $\frac{5}{6}$ of this as the true length, or $2l$.

H is given.

Substitute these values in the general formula (2) and calculate M.

Example.—Enter results thus:

Position of Magnet.	r	r_1	d	δ_1	δ_2	δ
East of needle............	30.0	42.6	36.3	2.32	2.35	2.33
Reversed............	......	.....	......	2.34	2.34	2.34
West of needle............	30.0	42.6	36.3	2.32	2.34	2.33
Reversed............	......	......	......	2.35	2.35	2.34
Mean values......................			36.3			2.33

$a = 5$ cm., $l = 5.12$, $H = .150$;

$$M = \frac{.150\{(36.30)^2 - (5.12)^2\}^2}{2 \times 36.30} \times \frac{2.33}{5}$$

$= 1606.$

Blank to be filled in by student.

Position of Magnet.	r	r_1	d	δ_1	δ_2	δ
East of needle............						
Reversed............						
West of needle............						
Reversed............						
Mean values.......						

$a =$ $l =$ $H =$

$M =$

$=$

46. TO DETERMINE THE MOMENT OF A MAGNET BY MEANS OF THE TORSION BALANCE.

References.—S. Thompson, p. 119; Barker, p. 539; Anthony and Brackett, p. 121; Carhart, pt. II. p. 161.

Apparatus Required.—A Coulomb balance provided with a graduated torsion-head and lower circle; half a meter of fine wire for the suspension; a long cylindrical brass bar; a long cylindrical magnet; a compass; a watch; a micrometer gauge; a centimeter scale.

Theory of Experiment.—If a magnet of moment M be suspended horizontally, and deflected through an angle θ from the meridian of a magnetic field whose horizontal component is H, by N turns of a suspending wire having a torsion couple T per unit angle, the condition of equilibrium is

$$2\pi NT = MH \sin \theta. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If a non-magnetic bar of known moment of inertia be oscillated in the stirrup carried by the suspension wire, the value of T can be found, since

$$T = \pi^2 n^2 K, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where n represents transits per sec., and K the moment of inertia. To compare the moments of two magnets by this method, it is only necessary to observe the turns of the torsion-head required to deflect them through the same angle, when the moments will be to each other as the turns of the torsion-head.

Practical Directions.—Weigh and measure the brass bar to the second decimal place.

Set it in the stirrup and level the instrument so that the bar may swing freely all round.

Set it oscillating through 20 or 30 degrees, and observe the time of twenty transits past the middle point of its swing.

Denoting the time by t,

$$n = \frac{20}{t}.$$

Calculate K from formula

$$K = m\left(\frac{l^2}{12} + \frac{c^2}{16}\right),$$

m, l, and c being respectively the weight, length, and diameter of the brass bar.

Calculate T from formula (2).

Having brought the brass bar to rest, turn the torsion-head till the brass bar lies parallel to the direction of the meridian as determined by the compass.

Note through which diametrically opposite graduations on the lower circle the meridian passes.

Remove the bar and replace it by the magnet, being careful to have the N pole pointing north.

Twist the torsion-head so that the magnet is deflected through an angle of 60° or 70° (θ).

Read the whole and fractional turns of torsion-head.

Bring the bar back to the meridian and deflect it through the same angle, θ, on the other side.

Let p denote mean turns of torsion-head.

Then N of formula (1) is equal to $\left(p - \frac{\theta}{360}\right)$, since the magnet is deflected through the angle θ.

The value of H is supposed to be known.

Calculate the moment of the given magnet from the formula (1).

Precaution.—As the magnet nears the east and west position, be careful to keep it from swinging widely. If allowed to pass the above position, it will swing completely around.

Example.—Enter results thus:

BRASS ROD.

W	l	c	K	n	T
37.36	18.14	.560	1025	.0311	9.77

MAGNET.

$H = .149.$

θ	Turns of Torsion-head.		Mean p.	$N = p - \frac{\theta}{360}$.	M
	Right	Left			
80°.0	2.48	2.46	2.47	2.25	941

Blanks to be filled in by student.

BRASS ROD.

W	l	c	K	n	T

MAGNET.

$H =$

θ	Turns of Torsion-head.	Mean p.	$N = p - \frac{\theta}{360}$.	M

47. TO DETERMINE THE HORIZONTAL INTENSITY OF THE EARTH'S MAGNETIC FIELD BY THE MAGNETOMETER METHOD.

References.—S. Thompson, pp. 121 and 124; Watson, p. 615; Ames, p. 355; Nichols and Franklin, p. 23; Anthony and Brackett, p. 268.

Apparatus Required.—A permanent bar magnet (preferably cylindrical); a delicate mirror magnetometer with silk-fibre suspension and provided with a long centimeter scale; a telescope for reading deflections of the magnetometer-needle.

Theory of Experiment.—Let the middle of a permanent magnet of moment M be brought up, in the end-on position, to a distance r from the centre of a delicately suspended magnetic needle. Let the needle be deflected in consequence through an angle θ. Then, if H_0 be the value of the controlling force on the needle (in this case the horizontal intensity of the earth's field),

$$M = \frac{1}{2}\frac{H_0(r^2 - l^2)^2 \tan\theta}{r} \quad . \quad . \quad . \quad . \quad (1)$$

(see page 140), where l is one-half the magnetic length of the deflecting magnet. If now the deflecting magnet be suspended by a fine thread and allowed to oscillate freely under the action of the earth's field, then

$$MH_0 = \pi^2 n^2 K, \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where K is the moment of inertia of the magnet, and n the number of transits per second.

Solving between (1) and (2), we get for H_0

$$H_0 = \frac{\pi n}{r^2 - l^2}\sqrt{\frac{2Kr}{\tan\theta}}. \quad . \quad . \quad . \quad . \quad (3)$$

The tangent of the angle of deflection is $\frac{\delta}{2D}$, where D is the distance from suspended needle to the scale of the telescope, and δ the mean deflection of the needle.

Hence $$H_0 = \frac{2\pi n}{r^2 - l^2}\sqrt{\frac{rKD}{\delta}}. \quad . \quad . \quad . \quad . \quad (4)$$

Practical Directions.—Level the magnetometer so that the needle swings freely. Set the scale east and west by means of the clamps provided. The centre of the scale should be under the needle approximately.

Set up the telescope and scale at a scale distance D of about one meter. Adjust the telescope to point on the mirror, and while looking along the telescope, adjust the scale up or down until an image of it is seen.

Focus the telescope on the image of the scale in the mirror.

Adjust the telescope and scale till the zero reading is opposite the vertical cross.

Turn the scale of the telescope until its ends are equidistant from the needle.

Place the deflecting magnet in its stirrup on the scale provided for it and bring it up to a distance r, so that a deflection of 100 mm. or so is obtained.

Read the deflection, δ_1, to $\frac{1}{10}$ mm. Reverse the magnet, being careful to have the centre of the magnet at the same distance, r, as before.

Read deflection, δ_2.

Transfer the magnet to the other side of the magnetometer at the same distance, r, and obtain similarly the readings, δ_3, δ_4.

A mean of the four readings should be taken. Hence the data for formula (1).

Now unhook the needle from the magnetometer and replace it by the deflecting magnet (a small aluminium wire stirrup is convenient for supporting the magnet).

Set the magnet swinging and count the time of fifty transits.

If t be the time, then $n = \frac{50}{t}$.

Measure the length, l, and diameter, c, of the magnet with a pair of micrometer calipers.

Weigh the magnet to $\frac{1}{10}$ milligram, denoting the weight by w.

Calculate K from the formula for cylindrical bars:

$$K = w\left(\frac{l^2}{12} + \frac{c^2}{16}\right).$$

Calculate the value of the earth's horizontal field from formula (4).

Example.—Enter results thus:

MAGNETOMETER OBSERVATIONS.

Deflections, mm.				Mean δ	D	r
δ_1	δ_2	δ_3	δ_4			
146	146	149	140	145	66.75	40

OBSERVATIONS ON MAGNET.

Weight.	Length.	Diameter.	n.
13.545	10.2	0.48	0.26

$$K = 13.545 \left\{ \frac{(10.2)^2}{12} + \frac{(0.48)^2}{16} \right\} = 117.6.$$

$$H_0 = \frac{2 \times 3.1416 \times 0.26}{(40)^2 - (4.5)^2} \sqrt{\frac{117.6 \times 66.75 \times 40}{14.5}} = .1522.$$

Blanks to be filled in by student.

MAGNETOMETER OBSERVATIONS.

Deflections, mm.				Mean δ	D	r
δ_1	δ_2	δ_3	δ_4			

OBSERVATIONS ON MAGNET.

Weight.	Length.	Diameter.	n.

$K =$

$H_0 =$

48. TO DETERMINE THE EQUIVALENT LENGTH OF A MAGNET.

References.—As in Experiment 45.

Apparatus Required.—A compass-box; a magnetometer with telescope and scale or lamp and scale; a permanent magnet.

Theory of Experiment.—Suppose that a permanent magnet, AB, is placed at right angles to the magnetic meridian and with its centre at a perpendicular distance r from a small magnetic needle at C.

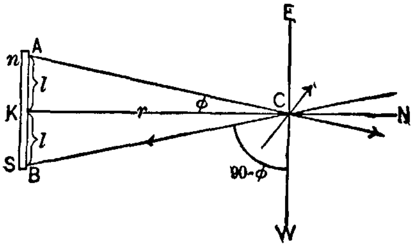

FIG. 82.

The force exerted by the N pole is

$$\frac{m}{l^2 + r^2},$$

and in the direction AC, m being the strength of the pole, and l half the length of the magnet.

The force exerted by the S pole is

$$\frac{m}{l^2 + r^2},$$

and in the direction CB.

The resultant of the two forces deflecting the needle is evidently along the line EW, and is equal to

$$\frac{2m}{l^2 + r^2} \sin \phi,$$

ϕ being the angle so marked in the figure.

$$\sin \phi = \frac{l}{\sqrt{l^2 + r^2}}.$$

Hence the resultant along EW is equal to

$$\frac{2ml}{(l^2 + r^2)^{\frac{3}{2}}}.$$

The moment of this force on the needle is

$$\frac{2ml}{(l^2 + r^2)^{\frac{3}{2}}} \cos \theta,$$

θ being the deflection of the needle.

This is equal to the earth's magnetic couple.

Hence $$\frac{2ml}{(l^2 + r^2)^{\frac{3}{2}}} \cos \theta = H \sin \theta,$$

or $$\frac{2ml}{(l^2 + r^2)^{\frac{3}{2}}} = H \tan \theta. \quad . \quad . \quad . \quad . \quad (1)$$

Suppose now the magnet to be brought to a position distant r_1 from the needle, the new deflection being θ_1.

Then $$\frac{2ml}{(l^2 + r_1^2)^{\frac{3}{2}}} = H \tan \theta_1. \quad . \quad . \quad . \quad . \quad (2)$$

Hence $$\frac{(l^2 + r_1^2)^{\frac{3}{2}}}{(l^2 + r^2)^{\frac{3}{2}}} = \frac{\tan \theta}{\tan \theta_1},$$

or
$$\frac{l^2 + r_1^2}{l^2 + r^2} = \left(\frac{\tan\theta}{\tan\theta_1}\right)^{\frac{2}{3}}.$$

Denoting $\left(\frac{\tan\theta}{\tan\theta_1}\right)^{\frac{2}{3}}$ by A,

and solving for l, we get

$$l = \sqrt{\frac{r^2 A - r_1^2}{1 - A}}. \quad . \quad . \quad . \quad . \quad (3)$$

If r, r_1, θ, θ_1, be observed, l can be calculated.

This position of the magnet with reference to the needle is called the "Broadside-on Position."

(2) Suppose the magnet to be placed at right angles to the meridian, and its axis in a line with the centre of the needle, as in Fig. 33.

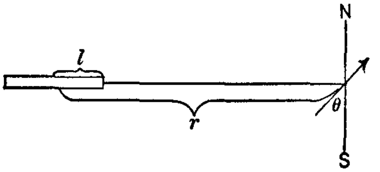

Fig. 33.

The conditions now are the same as in obtaining the moment of the magnet by deflection method.

Hence
$$\frac{4rlm}{(r^2 - l^2)^2} = H \tan\theta, \quad . \quad . \quad . \quad . \quad (4)$$

θ, r, and l having the same meaning as before.

For a position distant r_1 we also have

$$\frac{4r_1lm}{(r_1^2 - l^2)^2} = H \tan\theta_1. \quad . \quad . \quad . \quad . \quad (5)$$

Combining (4) and (5) and solving for l, we get

$$l = \sqrt{\frac{r^2 A - r_1^2}{A - 1}}, \quad . \quad . \quad . \quad . \quad . \quad (6)$$

A being equal in this case to

$$\left(\frac{r_1 \tan \theta}{r \tan \theta_1}\right)^{\frac{1}{2}}.$$

From (6) l can be again calculated if θ, θ, r, and r be observed.

This is known as the "End-on Position."

Practical Directions.—(1) *Broadside-on Position.*—Place the magnetometer in the magnetic meridian, and focus the telescope on the scale, the telescope being in a line east or west of the magnetometer.

Place the magnet, whose magnetic length is to be determined, at right angles to the meridian, having its centre and the centre of the needle in the meridian.

Adjust the distance till a deflection of nearly the whole scale is obtained.

Now reverse the magnet, and read again. The mean of the observations gives the true value for the deflection.

Denote the mean deflection by δ.

Measure the distance r. Allowance should be made for the width of the magnet.

Adjust the position of the magnet to another distance, r_1, so that a deflection of about half the previous one is obtained.

Read deflections and take the mean as before.

Measure r_1.

Measure the distance between the magnetometer and the telescope scale, denoting it by K.

Calculate $\left(\frac{\tan \theta}{\tan \theta_1}\right)^{\frac{2}{3}}$ or A,

remembering that $\tan 2\theta = \frac{\delta}{K}$, since the reflected angle is double the deflection of the mirror.

Substitute in formula (3), and calculate l.

(2) *End-on Position.*—Now place the magnet at right angles to the magnetic meridian, its axis being in a line with the centre of the needle.

Again adjust the distance r till a deflection of nearly the whole scale is obtained.

Read r and the deflection as before.

Reverse the magnet, and repeat the observations.

Adjust again for a deflection of about half the previous one, repeating the readings as above.

Calculate $\left(\frac{r_1}{r}\frac{\tan\theta}{\tan\theta_1}\right)^{\frac{1}{2}}$ or A.

Substitute in formula (6) and calculate l.

Example.—Enter results thus:

BROADSIDE-ON POSITION.

Position of Magnet.	r	δ	Mean δ	K	A	$2l$ Calculated.	Length of Bar.
N. to E. N. to W. N. to E. N. to W.	26.3 33.1	18.9 18.7 9.60 9.30	18.8 9.45	42	1.546	13.8	15.2
			END-ON POSITION.				
N. to E. N. to W. N. to E. N. to W.	40 50	11.3 12.3 6.0 5.65	11.8 5.82	41.3	1.580	12.7	15.2

Blank to be filled in by student.

BROADSIDE ON POSITION.

Position of Magnet.	r	δ	Mean δ	K	A	$2l$ Calculated.	Length of Bar.
N. to E. N. to W. N. to E. N. to W.							
END-ON POSITION.							
N. to E. N. to W. N. to E. N. to W.							

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49. TO DETERMINE THE VARIATIONS IN THE HORIZONTAL INTENSITY OF THE EARTH'S MAGNETIC FIELD BY MEANS OF THE COMPASS-BOX VARIOMETER.

References.—Kohlrausch's Physical Measurements, p. 257.

Apparatus Required.—Kohlrausch's compass-box variometer.

Theory of Experiment.—The variometer consists essentially of a permanent magnet and compass-box, the box being upon the top of an upright which passes through the centre of the magnet, the centre of the magnet and the needle having the same vertical axis. The magnet can be adjusted vertically and turned round its centre.

Suppose the magnet to be fixed with regard to its vertical motion, and to be turned round until its N pole points north. The corresponding pole of the needle in the compass-box will point directly south.

If now the magnet be turned through an angle θ, such that the needle lies in an east and west direction, we have

$$F \cos \theta = H_0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where F is the force due to the magnet, and H_0 the earth's horizontal component.

Now let the instrument be moved to another station, where H_1 is the earth's horizontal component.

If now the magnet be turned through the same angle, θ, from the meridian, the needle will take up a different position,

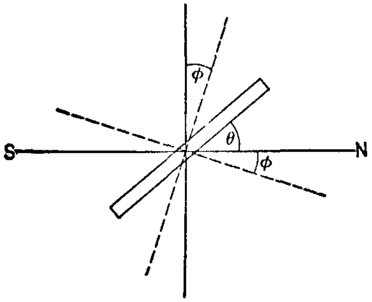

FIG. 34.

making an angle ϕ with the east and west direction (see dotted line in Fig. 34), unless H_1 be equal to H_0.

In this case we have

$$H_1 \cos \phi = F \cos(\theta \pm \phi),$$

or

$$H_1 = F(\cos \theta \mp \sin \theta \tan \phi). \quad . \quad . \quad . \quad (2)$$

Hence, combining (1) and (2), we have

$$H_1 = H_0(1 \mp \tan \theta \tan \phi). \quad . \quad . \quad . \quad (3)$$

From (3) H_1 can be calculated if H_0 be known.

If ϕ be small and measured in degrees,

$$\tan \phi = \frac{\phi \times \pi}{180} \text{ approx.,}$$

and formula (3) becomes

$$H_1 = \left\{ 1 \mp \phi \frac{\pi}{180} \tan \theta \right\} H_0. \quad . \quad . \quad . \quad (4)$$

Practical Directions.—(1) *Adjustments at Station of Reference*, H_0.—Carefully level the instrument. Set the zero of the scale carried by the magnet to one of the quadrant divisions on the fixed scale immediately below it. Lower the needle in the compass-box till it swings freely on its pivot.

Turn the whole instrument, compass-box, and magnet together, till the needle and magnet are parallel, the N pole of the magnet pointing north. The exact position is found by turning till the needle reads to the quadrant-point of the box.

Clamp the box to the stand.

Now turn the magnet until the needle is deflected just 90°, and set the stop provided against the magnet.

Reverse the needle by turning the magnet through an angle on the other side of the meridian until the needle is deflected 90° in the opposite direction.

Clamp the second stop.

(2) *Adjustments at Second Station*, H_1.—Take the instrument now to another station. Level as before. Adjust as before until the needle and magnet are in the meridian. Without moving the stops, turn the magnet successively to them and in each case read the difference of the deflection from 90°. Read in each case both ends of the needle, and take the mean of the four readings as ϕ.

Read the angle through which the magnet is turned from the meridian, and take the mean of the two readings as θ. H_1 will be greater or less than H_0 according as the needle in

the second station is deflected through an angle greater or less than 90°.

Precaution.—When once the magnet has been adjusted at the station of reference, do not alter its position vertically or move the stops, otherwise the work will have to be repeated.

Example.—Enter results thus:

Station.	θ	ϕ				Mean Value of ϕ	H
		Magnet East of Meridian.		Magnet West of Meridian.			
		N. End of Needle.	S. End of Needle.	N. End of Needle.	S. End of Needle.		
Reference	32° 30′	0	0	0	0	0	.150
2d	32° 50′	+ 3.5	+ 3.6	+ 3.5	+ 3.6	+ 3.55	.156
3d	Mean Value	+ 4.7	+ 4.7	+ 4.6	+ 4.6	+ 4.65	.158
4th	32° 40′	− 2.7	− 2.6	− 2.6	− 2.7	− 2.65	.145

Blank to be filled in by student.

Station.	θ	ϕ				Mean Value of ϕ	H
		Magnet East of Meridian.		Magnet West of Meridian.			
		N. End of Needle.	S. End of Needle.	N. End of Needle.	S. End of Needle.		
Reference							
2d							
3d							
4th							

ELECTRICITY.

50. TO COMPARE THE SINE AND TANGENT METHODS OF MEASURING CURRENTS.

References.—Knott, pt. II. p. 175; Watson, pp. 682–687; S. Thompson, pp. 201 and 202; Carhart, pt. II. p. 335; Barker, p. 775; Anthony and Brackett, p. 357; Ames, p. 365; Nichols and Franklin, p. 38; Hastings and Beach, pp. 415–417.

Apparatus Required.—A galvanometer suitable for both sine and tangent methods; a storage-battery; a resistance-box; a reversing-key.

Theory of Experiment.—If G be the strength of magnetic field at the centre of a galvanometer coil produced by a unit current flowing through the coil, then GC will be the strength of field for a current C, the lines of force being at right angles to the coil.

Suppose the coil of the galvanometer to be in the magnetic meridian, and the galvanometer needle so small that the field in which it acts may be considered uniform.

Let M be the magnetic moment of the needle.

Then if the needle be deflected through an angle θ by the current C,

$$GCM \cos \theta = HM \sin \theta, \quad . \quad . \quad . \quad . \quad (1)$$

where H is the earth's horizontal component.

Hence $$C = \frac{H}{G} \tan \theta = K \tan \theta. \quad . \quad . \quad . \quad (2)$$

If now, while the current C is still flowing, the coil of the galvanometer be turned round, following the needle, until the needle is again in equilibrium and in the same relative position to the coil as it was before the current was turned on, then

$$GCM = HM \sin \theta_1,$$

or $$C = K \sin \theta_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

θ_1 being the angle through which the coil must be turned to bring the needle to its original position in relation to the coil.

Hence, combining (2) and (3),

$$K \sin \theta_1 = K \tan \theta,$$

or $$\sin \theta_1 = \tan \theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Practical Directions.—Place the coil of the galvanometer accurately in the magnetic meridian. This may be done by placing the coil so that the needle reads zero on the compass-box scale.

Connect in series the galvanometer, a resistance-box, and the storage-battery, putting, however, in the galvanometer circuit a reversing-key so that the current can be reversed through the galvanometer. (See Fig. 35.)

In the figure $ABCD$ is the reversing-switch, G the galvanometer, R the resistance, B_1 the battery.

By connecting A to B, and C to D, the current flows through the galvanometer in one direction, and in the opposite by joining A to C, and B to D.

Before closing the circuit take out a plug from the resistance-box so that at least 50 ohms shall be in the circuit.

Close the circuit by means of the reversing-key, and adjust the resistance until a deflection of, say, 10° is obtained.

Reverse the current and read again.

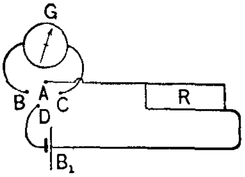

FIG. 35.

Both ends of the needle should be read each time to the $\frac{1}{10}$ of a degree.

Denote the angles by θ_1, θ_2, θ_3, θ_4, and the mean angle by θ.

With the current still flowing, turn the galvanometer coil round its vertical axis until the needle again reads zero. If the galvanometer be provided with a scale for measuring the angle through which the coil be turned, this angle can be read off directly.

If this be not the case, when the needle reads zero, open the circuit and read the angle through which it swings back.

Since in this case the needle comes back to the meridian, this angle will be the same as that through which the coil was turned.

Reverse the current, and repeat the operation as above.

Denote the angles by ϕ_1, ϕ_2, ϕ_3, ϕ_4, and the mean angle by ϕ.

Then by formula (4)

$$\tan \theta = \sin \phi.$$

Bring the coil back again to its original position, and adjust the resistance until the reading by the tangent method is approximately 15°, and take the corresponding sine reading.

Adjust again for 20°, 25°, 30°, and 35°, comparing the tangents of these angles with the sines of corresponding angles by sine method.

Record the resistance R used in each case.

Example.—Enter results thus:

R	Mean θ.	Mean ϕ.	tan θ.	sin ϕ.	Difference.
70	10°.25	10°.5	.1808	.1822	+ .0014
50	13 .50	13 .8	.2400	.2385	– .0015
30	19 .75	21 .1	.3590	.3599	+ .0009
20	26 .50	29 .9	.4985	.4984	– .0001
10	39 .75	56 .5	.8317	.8338	+ .0021

Blank to be filled in by student.

R	Mean θ.	Mean ϕ.	tan θ	sin ϕ.	Difference.

51. TO DETERMINE THE ABSOLUTE MEASURE OF AN ELECTRIC CURRENT IN AN INCANDESCENT LAMP.

References.—As in Experiment 50.

Apparatus Required.—A tangent galvanometer (the coil of which can be measured); a plug for connecting with the lighting circuit; a portable incandescent-lamp stand; a reversing-switch.

Theory of Experiment.—If G be the strength of the magnetic field at the centre of the galvanometer coil due to a unit current, C the current flowing, H the earth's horizontal component, θ the deflection of the needle, then, if the coil be in the magnetic meridian, we have

$$C = \frac{H}{G} \tan \theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If r be the mean radius of the coil, and n the number of turns of wire,

$$G = \frac{2\pi r n}{r^2}.$$

Substituting in (1), we get

$$C = \frac{Hr \tan \theta}{2\pi n}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If the current be measured in amperes,

$$C = \frac{10Hr \tan \theta}{2\pi n}. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If H be known, θ observed, and r measured, C can be calculated.

Practical Directions.—Place the galvanometer in the magnetic meridian.

Connect in series the galvanometer, the reversing-switch, the lamp in the portable lamp-stand and the lighting-circuit. This is assuming of course that the current is a direct one. A suitable form of lamp-stand can be made by connecting two lamp-sockets on a board so that the lamps can be used singly or in parallel.

Put a lamp in one socket, and turn on the current.

Read the deflection of the needle at both ends to $\frac{1}{10}$ of a degree, denoting the values by δ and δ_1.

Reverse the current and read as before, δ_3 and δ_4.

Then $$\theta = \frac{\delta + \delta_1 + \delta_3 + \delta_4}{4}.$$

Measure by means of a pair of calipers the diameter of the coil. To do this at least three different diameters for both the inside and outside of the coil should be measured, and the mean taken.

This mean diameter divided by 2 gives r, the radius of the coil. The value of H can be found on a chart in the laboratory.

Substitute the values in the formula

$$C = \frac{10 Hr \tan \theta}{2\pi n}.$$

Measure the current through one 16-C.P. lamp, then through two 16-C.P. lamps in parallel.

Then measure the current through two 32-C. P. lamps in parallel, and also through one 32-C.P. lamp.

Finally through one 32-C.P. lamp and one 16-C.P. lamp in parallel.

Precautions.—Be careful not to short-circuit the lighting circuit. Make all the connections and be sure they are correct before connecting the plug with the lamp-socket.

Example.—Enter results thus:

	δ°	δ_1°	δ_3°	δ_4°	θ°	Outside Diam.	Inside Diam.	R.	Current Amps.
One 16-C.P. lamp...	6.5	6.5							
Reversed			6.8	6.7	6.7	34.8	33.6		.48
Two 16-C.P. lamps..	14.5	14.5						17.1	
Reversed			14.9	14.9	14.7	34.8	33.6		1.03
One 32-C.P. lamp....	20	20							
Reversed			20.2	20.2	20.1	34.8	33.6		1.49
Two 32 C.P. lamps..	36	36							
Reversed			37	37	36.5	34.8	33.6		3.02

Blank to be filled in by student.

	δ°	δ_1°	δ_3°	δ_4°	θ°	Outside Diam.	Inside Diam.	R.	Current Amps.
One 16-C.P. lamp...									
Reversed........									
Two 16-C P. lamps..									
Reversed									
One 32-C.P. lamp...									
Reversed.......									
Two 32 C.P. lamps .									
Reversed...........									

52. TO DETERMINE THE ELECTRO-CHEMICAL EQUIVALENT OF HYDROGEN.

References.—Knott, pt. II. pp. 201–206; Hastings and Beach, pp. 396–400; Watson, pp. 786–788; S. Thompson, pp. 224–228; Nichols and Franklin, pp. 67–69; Ames, pp. 317–322; Anthony and Brackett, pp. 323–329; Barker, pp. 741–746; Carhart, pt. II. pp. 255–260.

Apparatus Required.—A tangent galvanometer, the coil of which can be measured; a gas voltameter; a resistance-box; a four-volt storage-battery or other source of constant current; a reversing-key; a thermometer.

Theory of Experiment.—The electro-chemical equivalent of a substance is the mass of the substance deposited by the passage of a unit quantity of electricity through an electrolyte in which the substance is an ion.

If the gas voltameter, the battery, and the tangent galvanometer be connected in series, then the current flowing through the circuit is given by the equation

$$C = \frac{H}{G} \tan \theta = \frac{Hr \tan \theta}{2\pi n}, \quad . \quad . \quad . \quad . \quad (1)$$

where θ is the deflection, H the earth's horizontal component, r the mean radius of the coil, and n the number of turns of wire in the coil.

If now, in a time t'', the quantity of hydrogen deposited by the current, supposed constant, be m, and the electro-chemical equivalent be denoted by e, then

$$Ct'' = \frac{m}{e};$$

therefore

$$e = \frac{m}{Ct''}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Combining (1) and (2), we have

$$e = \frac{2\pi nm}{Hrt'' \tan \theta}. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If now m, the mass deposited in a given time, be measured, H be known, and θ observed, e can be calculated, r and n being supposed known or measurable quantities.

Practical Directions.—Connect in series the galvanometer, the gas voltameter, the battery, the resistance-box, and the reversing-switch. The switch should be in the galvanometer circuit only, as in Experiment 50, Fig. 35.

Unplug from the resistance-box 100 ohms.

Set the galvanometer coil carefully in the meridian.

Close the switch, and adjust the resistance until a suitable deflection is obtained, viz., 45° to 50°.

Open the switch and let the hydrogen and oxygen escape.

Now close the key again, and take accurately the time of starting.

Reverse the current every two minutes, and take readings for both ends of the needle. Denote the first readings by θ_1, θ_2, and the readings when current is reversed by θ_3, θ_4.

The mean of the deflections observed gives the value of θ.

Let the current flow until the tube containing hydrogen is nearly full.

Take accurately the time at which the current is turned off.

Measure r, the radius of the galvanometer coil.

Read the volume of hydrogen in the tube of the voltameter.

Take the temperature, t, of the solution in the voltameter.

Read from the chart in the room the aqueous vapor pressure for temperature, t.

Read the barometric pressure, correcting for temperature.

Read the difference of head between the hydrogen and the water in the open tube.

If the hydrogen tube of the voltameter be not graduated, the volume can readily be obtained as follows:

Let the oxygen escape from the oxygen tube.

By means of a pipette or siphon, take the solution out of the open tube down to some fixed point just above the upper end of the hydrogen tube. It is convenient to have a permanent mark on the open or central tube for the purpose.

Now let the hydrogen escape.

Fill a graduated burette with some of the solution.

Let the solution flow from the burette into the voltameter until the hydrogen tube is just filled.

Close the cock of the hydrogen tube, care being taken not to let any solution escape through it.

Fill the central tube exactly to the marked point.

Read the burette.

The volume emptied into the voltameter will be equal to the volume of hydrogen in the tube at the beginning, v_1.

In this case h should be measured from the marked point to the surface of the liquid in the hydrogen tube just before it is allowed to escape.

To find m.—To find the value of m, we must calculate the volume under standard pressure and at 0° C., and multiply by the density of hydrogen, .0000896.

If v_0 be the volume of the hydrogen at standard temperature, 0° C. or 272.5 of the absolute scale, and under standard pressure, 76 cm., v_1 the observed volume at temperature t or 272.5 + t of the absolute scale, and under a pressure P, then we have the relation

$$\frac{v_0 \times 76}{272.5} = \frac{v_1 \times P}{272.5 + t},$$

or

$$v_0 = \frac{v_1 \times P \times 272.5}{76\,(272.5 + t)}. \quad . \quad . \quad . \quad . \qquad (4)$$

The pressure P is made up of three parts: first, the barometric pressure B; second, the pressure due to the head of water in the voltameter; third, the pressure due to the presence of aqueous vapor in the tube containing hydrogen.

If h be the difference in head in the voltameter, then this correction reduced to centimeters of mercury is $\frac{hs}{13.596}$, where s is the density of the solution used, 13.596 being that

of mercury. If the solution be 15 per cent. sulphuric acid, $s = 1.1$ approximately. Assuming this to be the case, we have

$$v_0 = \frac{v_1\left(B + \frac{h \times 1.1}{13.596} - a\right) \times 272.5}{76(272.5 + t)}, \quad . \quad . \quad (5)$$

and therefore

$$m = \frac{v_1\left(B + \frac{h \times 1.1}{13.596} - a\right) \times 272.5 \times .0000896}{76(272.5 + t)}, \quad . \quad (6)$$

a being the aqueous vapor pressure which can be found for the temperature t from a chart in the laboratory, or from a book of tables, and .0000896 the density of hydrogen, that is, the mass per cubic centimeter.

Substituting this value for m in equation (3), we obtain e_1.

Example.—Enter results thus:

$H = .1558. \quad n = 30.$

θ_1	θ_2	θ_3	θ_4
46.7	46.2	46.9	46.5
46.4	46.0	46.5	46.4
46.5	46.3	46.7	46.5
46.4	46.2	46.5	46.5
46.3	46.1	46.5	46.4
46.4	46.1	46.5	46.4
46.3	45.9	46.4	46.4
46.5	46.1	46.4	46.3
46.3	46.3	46.3	46.5
46.5	46.3	46.5	46.5
Mean value θ			46.4

Time of Starting.	Time of Finishing.	t''	B (corrected).
2.45	3.25	2400	76.02
r	h	t	α
15.1	30.5	18.0	1.54
v_1	v_0	M	$\frac{e}{10^7}$
38.3	36.37	.003258	104

Blank to be filled in by student.

θ_1	θ_2	θ_3	θ_4	Time of Starting.	Time of Finishing.	t''	B (corrected.)
				r	h	t	α
				v_1	v_0	M	$\frac{e}{10^7}$
Mean value θ.........							

53. TO COMPARE THE ELECTROCHEMICAL EQUIVALENTS OF COPPER AND HYDROGEN.

References.—References as in Experiment 52.

Apparatus Required.—A copper voltameter; a gas voltameter; a four-volt storage-battery or a plug for the lighting-circuit; a variable resistance; a contact key.

Theory of Experiment.—If the source of current be connected in series with a gas voltameter and a copper voltameter, the ratio of the electrochemical equivalents may be determined by determining the mass of hydrogen and the mass of copper deposited in a given time, and dividing the one by the other. If m be the mass of hydrogen, m_1 the mass of copper, then

$$R = \frac{m_1}{m}. \quad \ldots \ldots \quad (1)$$

Practical Directions.—Connect in series a copper voltameter containing a solution of copper sulphate; a gas voltameter containing a 15-per-cent solution of sulphuric acid; the battery (or lighting-circuit); the adjustable resistance; the contact-key.

If the lighting-circuit be used, a 16-candle-power lamp should be in the circuit.

Take out the copper plate upon which the copper is to be deposited, and thoroughly clean it. This may be done by washing in a nitric-acid solution or by rubbing, when wet, with emery paper and thoroughly washing in pure water. Dry the plate thoroughly and weigh to a milligramme.

Connect this plate to the negative pole of the circuit.

The negative pole may be found by connecting the source of current to the gas voltameter alone and observing the tube in which the hydrogen is deposited.

In doing this, however, be sure to have a resistance of not less than 100 ohms in the circuit (a 16-C. P. lamp or other resistance) if the lighting-circuit be used.

See that the tubes of the gas voltameter are full of the solution.

Close the circuit and let it flow until the hydrogen tube of the gas voltameter is nearly full of hydrogen.

To find m_1.—Take out the copper plate upon which the deposit has been made, rinse in pure water, and dry and weigh as before.

If w be the weight before and w_1 after the deposit has been made, then

$$m_1 = w_1 - w.$$

To find m, the same observations must be made as in the last experiment. The terms having the same meaning as in that case,

$$m = \frac{v_1\left(B + \dfrac{h \times 1.1}{13.596} - a\right) \times 272.5 \times .0000896}{76 \times (272.5 + t)}.$$

Precautions.—(1) Be sure not to short-circuit the lighting-circuit through the gas voltameter.

(2) The cleaned copper plate must be connected to the negative pole of the source of current.

(3) Thoroughly clean and dry the plate each time before weighing it.

Example.—Enter results thus:

w	w_1	m_1	B	t
75.063	75.160	.097	76.02	18°
a	h	v_1	m	R
1.54	35.7	36.5	.00312	31.10

Blank to be filled in by student.

w	w_1	m_1	B	t
a	h	v_1	m	R

54. TO DETERMINE THE HORIZONTAL COMPONENT OF THE EARTH'S MAGNETIC FORCE.

References.—Same as in Experiments 51 and 52.

Apparatus Required.—A tangent galvanometer, the coil of which can be measured; a copper voltameter; a reversing-key; a storage battery; an adjustable resistance.

Theory of Experiment.—We have seen that if a current flow through the coil of a tangent galvanometer, producing a deflection, the galvanometer coil being in the meridian, the current is given by the equation

$$C = \frac{H}{G} \tan\theta = \frac{Hr \tan\theta}{2\pi n}.$$

Hence

$$H = \frac{2\pi n \cot\theta \, . \, C}{r}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

n being the number of turns of wire in the coil, and r its radius.

If now C be measured by means of a copper voltameter, H can be calculated.

Let M denote the mass of copper deposited by the passage of the current through the voltameter, e the electro-chemical equivalent of copper, t'' the time in seconds during which the current flows; then as before (Experiment 52),

$$Ct'' = \frac{M}{e}, \quad \text{or} \quad C = \frac{M}{et''}. \quad . \quad . \quad . \quad (2)$$

Hence, combining (1) and (2),

$$H = \frac{2\pi n \cot\theta \times M}{ert''}. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Practical Directions.—Connect in series the tangent galvanometer, the battery, the copper voltameter, the reversing-key, and an adjustable resistance.

Put the reversing-key in the galvanometer circuit only (Fig. 35).

Close the circuit and adjust the resistance until a suitable deflection, about 45°, is obtained, the coil of the galvanometer being in the meridian.

A meter or two of bare German-silver wire, No. 20, makes a suitable resistance and can be adjusted at one terminal of the battery.

Now open the circuit and take out the plate upon which the deposit is to be made. Clean, dry and weigh, as in the last experiment, and restore the plate to its place again.

Be sure that the negative pole of the battery is connected to the clean plate, otherwise copper will be taken off instead of being deposited upon it.

Close the circuit, taking accurately the time of closing.

Let the current flow for 30 minutes or more.

Take accurately the time when the current is turned off.

While the current is on, reverse every two minutes and read deflections, reading always, if possible, both ends of the needle, denoting the four readings by θ_1, θ_2, θ_3, θ_4.

The mean of these gives the true value of θ.

Unless a rapidly reversing commutator is used, the time of each reversal should be taken and allowed for.

Take again from the voltameter the plate upon which the copper deposit has been made.

Wash the plate by letting pure water flow gently over it, or by rinsing in a 10 per cent. solution of sulphuric acid.

Dry as before by holding it *near* a Bunsen flame.

Weigh the plate.

The difference between the two weights is the copper de-

posited. Measure the radius of the coil as in previous experiment.

The electro-chemical equivalent of copper, E, is .003286.

Example.—Enter results thus:

<table>
<tr><th>θ_1</th><th>θ_2</th><th>θ_3</th><th>θ_4</th><th>W</th><th>Time of Starting.</th><th>n</th></tr>
<tr><td rowspan="4">46.5
46.2
46.3
46.2
46.1
46.2
46.1
46.3
46.1</td><td rowspan="4">46.0
45.8
46 1
46.0
45.9
45.9
45.7
45.9
46.1</td><td rowspan="4">46 7
46 3
46.5
46.5
46.3
46.3
46.2
46.1
46.3</td><td rowspan="4">46.3
46.2
46.3
46.3
46.2
46.2
46.3
46.3
46.1</td><td>250.700</td><td>2.35</td><td>30</td></tr>
<tr><td>W_1</td><td>Time of Finishing.</td><td>r</td></tr>
<tr><td>250.748</td><td>30.5</td><td>15.1</td></tr>
<tr><td>M</td><td>t''</td><td>H</td></tr>
<tr><td colspan="3">Mean value, θ.... ......</td><td>46.2</td><td>.048</td><td>1200</td><td>.147</td></tr>
</table>

Blank to be filled in by student.

<table>
<tr><th>θ_1</th><th>θ_2</th><th>θ_3</th><th>θ_4</th><th>W</th><th>Time of Starting.</th><th>n</th></tr>
<tr><td rowspan="4"></td><td rowspan="4"></td><td rowspan="4"></td><td rowspan="4"></td><td></td><td></td><td></td></tr>
<tr><td>W_1</td><td>Time of Finishing.</td><td>r</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>M</td><td>t''</td><td>H</td></tr>
<tr><td colspan="3">Mean value, θ ....</td><td></td><td></td><td></td><td></td></tr>
</table>

55. TO DETERMINE THE REDUCTION FACTOR OF A GALVANOMETER.

References.—As in Experiment 54.

Apparatus Required.—A tangent galvanometer; a gas or copper voltameter; a storage-battery or plug for the lighting circuit; an adjustable resistance capable of carrying one-fifth of an ampere; a reversing-switch.

Theory of Experiment.—The theory of this experiment is exactly the same as the last, the only difference being, that in this case, since the value G cannot be directly measured, the value $\frac{H}{G}$, the "reduction factor," is obtained.

Since $$C = \frac{H}{G} \tan \theta = K \tan \theta,$$

$$K = C \cdot \cot \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Practical Directions.—The connections and observations are exactly as in the last experiment.

If a gas voltameter be used, observations similar to those in finding the electro-chemical equivalent of hydrogen must be made.

If the current be taken from the lighting circuit, a lamp should always be in series with from 500 to 900 ohms resistance. An ordinary resistance-box is not suitable, as the coils are liable to burn out. A coil made from No. 24 or 25 German-silver wire serves the purpose.

In the case recorded below a gas voltameter was used.

Example.—Enter results thus:

θ Mean.	t''	B	h	t	a	v	C	K
46° 18′	3840	76.75	35.0	17	1.44	38.28	.0837	.080

Blank to be filled in by student.

θ Mean.	t''	B	h	t	a	v	C	K

56. TO PROVE OHM'S LAW, $C = \frac{E}{R}$.

References.—Knott, pt. II. pp. 184–187; Watson, p. 688; Barker, p. 699; S. Thompson, pp. 175 and 397; Hastings and Beach, p. 395; Nichols and Franklin, vol. II. p. 54; Anthony and Brackett, p. 317; Ames, p. 333.

Apparatus Required.—A tangent galvanometer (preferably one sensitive to fairly small currents); a storage battery; a resistance-box; a reversing-switch.

Theory of Experiment.—If the galvanometer, the resistance-box, and the source of current be connected in series, then the relation between current and deflection is given by the equation $C = K \tan \theta$.

Further, by Ohm's Law,

$$C = \frac{E}{R}.$$

Hence $$K \tan \theta = \frac{E}{R}.$$

K is constant, E is constant if a storage battery be used;

hence $$R = \frac{\alpha}{\tan \theta},$$

where α is a constant.

Suppose a series of values of θ be observed for different values of R, and a curve plotted, with R for abscissas and $\frac{1}{\tan \theta}$ for ordinates; then if $C = \frac{E}{R}$, the curve will be a straight line.

Practical Directions.—Connect in series the galvanometer, the storage battery, the resistance-box, and the reversing-key, the galvanometer coil being carefully placed in the meridian.

Unplug a large resistance from the box.

Close the circuit, and adjust the resistance until a deflection of about 10° is obtained.

Read the resistance, R, and the deflection, θ_1.

Reverse the current, and read again, θ_2.

The mean value of the deflections gives θ.

Diminish the resistance in the circuit until the deflection is about 15°, reverse and read as before.

Change the resistance, obtaining deflections as nearly as possible to 20°, 25°, 30°, 35°, 40°, 45°, 50°, 55°, reading corresponding values for the resistances.

Plot a curve for $\frac{1}{\tan \theta}$ and R as indicated above.

It will be noticed that the line does not pass through the origin. This is because the resistance of the galvanometer and battery has been neglected in plotting.

If the line be produced to cut the axis of abscissas, the negative abscissa will obviously be the resistance of the galvanometer and battery.

Precautions.—Before connecting in the battery, be sure to unplug a large resistance from the box.

Never have less than 20 ohms in the circuit, unless the resistance-box is known to be suited for currents used.

Example.—Enter results thus:

θ_1	θ_2	θ	tan θ	$\frac{1}{\tan\theta}$	R
16° 45′	16° 45′	16° 45′	.3009	3.325	70
22° 00′	23° 00′	22° 30′	.4142	2.420	50
27° 50′	28° 50′	28° 20′	.5391	1.880	40
34° 30′	34° 30′	34° 30′	.6873	1.456	30
45° 00′	45° 25′	45° 12′	1.0070	0.992	20

Blank to be filled in by student.

θ_1	θ_2	θ	tan θ	$\frac{1}{\tan\theta}$	R

---

57. COMPARISON OF ELECTRICAL RESISTANCES BY MEANS OF A SINE OR TANGENT GALVANOMETER.

References.—Knott, p. 197; Watson, p. 688; Barker, p. 700; Hastings and Beach, pp. 425–429; Nichols and Franklin, vol. II. p. 91; S. Thompson, p. 413; Anthony and Brackett, pp. 319 and 360; Ames, pp. 333–337.

Apparatus Required.—A sine or tangent galvanometer; a resistance-box; a storage battery; a reversing-switch, resistances to be measured.

Theory of Experiment.—If a galvanometer, a known resistance R, an unknown resistance X, and a battery, be connected in series, from Ohm's Law,

$$C = \frac{E}{B + G + R + X} = K \tan \theta \quad . \quad . \quad (1)$$

if the galvanometer be a tangent galvanometer,

or $$= K \sin \theta$$

if it be a sine galvanometer, B and G being the resistance of the battery and galvanometer respectively.

If now the resistance X be removed from the circuit, and R adjusted to R_1 so as to give the same deflection as before,

then $$B + G + R + X = B + G + R_1,$$

or $$X = R_1 - R. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

It is usually impossible to adjust the resistance R_1 so as to get the same deflection exactly.

If R_1 be, therefore, a resistance which gives a deflection θ_1,

then $$C_1 = \frac{E}{B + G + R_1} = K \tan \theta_1 . \quad . \quad . \quad . \quad (3)$$

Hence $$\frac{\tan \theta}{\tan \theta_1} = \frac{B + G + R_1}{B + G + R + X},$$

combining (1) and (3).

If $B + G$, R and R_1 be known, θ and θ_1 observed, X can be calculated.

If $B + G$ be not known, they can be first calculated thus:

Using equation (3),

$$\frac{E}{B + G + R_1} = K \tan \theta_1.$$

Changing R_1 to R_2, we get

$$\frac{E}{B + G + R_2} = K \tan \theta_2.$$

Hence
$$\frac{B + G + R_2}{B + G + R_1} = \frac{\tan \theta_1}{\tan \theta_2},$$

or, denoting $B + G$ by Y,

$$\frac{Y + R_2}{Y + R_1} = \frac{\tan \theta_1}{\tan \theta_2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

from which Y can at once be calculated.

Practical Directions.—Connect in series the resistance-box, the battery, the galvanometer, and a reversing-switch.

Before closing the circuit unplug from the box a large resistance.

Set the galvanometer coil accurately in the meridian.

Close the circuit and adjust the resistance until the deflection is about 30°. Denote reading by δ.

Reverse the current and read again, δ_1, taking the mean as θ.

Adjust the resistance again until a deflection of about 60° is obtained.

Reverse and read as before, δ_2, δ_3, taking the mean as θ_1.

From these observations calculate $B + G$.

Now put in the unknown resistance X, and adjust the resistance in the box until the deflection is again about 60°.

Use this observation with the first (30°) to calculate the value of X.

Repeat the operation for three or four different resistances.

If a sine galvanometer be used, substitute sin θ for tan θ in all the calculations, but the deflections must be taken in accordance with the sine method, see Experiment 50.

Example.—Enter results thus:

δ	δ_1	θ	δ_2	δ_3	θ_1	R	R_1	$B+G$	X
30.	31.3	30.7	63.	61.	62.	30	9	.68	
11.4	12.0	11.7	11.2	11.8	11.5	30	89		57.0
26.5	26.9	26.7	27.0	27.4	27.2	30	37		7.7

Blank to be filled in by student.

δ	δ_1	θ	δ_2	δ_3	θ_1	R	R_1	$B+G$	X

58. TO MEASURE ELECTRICAL RESISTANCES BY A B. A. WIRE BRIDGE.

References.—Knott, p. 198; Watson, pp. 694 and 695; S. Thompson, p. 415; Hastings and Beach, pp. 433 and 434; Nichols and Franklin, vol. II. p. 93; Anthony and Brackett, pp. 361 and 362; Ames, p. 337.

Apparatus Required.—A B. A. bridge; a battery; a low-resistance galvanometer; a resistance-box; a contact-key; resistances to be measured.

Theory of Experiment.—If an electric current C flow through a conductor R, then, by Ohm's law,

$$C = \frac{E}{R},$$

E being the E. M. F. of the source of current.

Hence the E. M. F. between any two points on the conductor is proportional to the resistance between these points.

If a stretched wire be connected in parallel with two resistances which are in series, and a current flow through the whole, as per diagram, AK being the wire, AC and CK

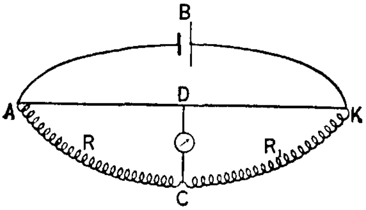

FIG. 36.

the resistances R and R_1 respectively, and B the battery, then the difference of potential between the points A and K is the same whether we consider the path ACK or ADK.

Hence when $$\frac{AK}{AD} = \frac{R + R_1}{R},$$

or $$\frac{DK}{AD} = \frac{R_1}{R}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

the points D and C are at the same potential, and therefore no current will flow through a galvanometer connected to D and C.

In this case $$R_1 = \frac{\text{Resistance of } DK}{\text{Resistance of } AD} \times R.$$

If R be a known resistance, AK a uniform wire, and a point D be found in it so that no deflection of a galvanometer takes place when D and C are connected through it, then if the lengths AD and DK be measured, R_1 can be calculated.

Practical Directions.—The B. A. bridge consists of a uniform wire AB stretched against a centimeter scale so that the lengths of the segments of the wire can be read off at once. It is provided with terminals for connecting in the resistances to be measured, the standard resistance, and the battery.

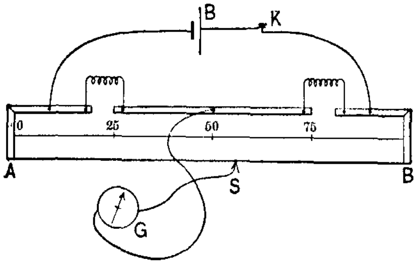

Fig. 37.

Connect the standard resistance and the unknown resistance in the bridge, as in Fig. 37.

Connect the battery, B, through a contact-key, K, to the terminals provided for the purpose.

Connect the point between the two resistances to the galvanometer, and through the other terminal of galvanometer, G, to the sliding contact, S.

Close the battery circuit first and then press lightly the sliding contact on the wire: the galvanometer will be deflected.

Adjust the position of the contact, repeating the operation until no deflection is obtained.

The standard resistance should be a one-ohm box divided in tenths, and the resistance should be adjusted so that the balance-point, *S*, is near the middle of the wire.

Having found the balance-point, read the lengths of the segments of the wire.

Calculate R_1.

The method is only suitable for measuring small resistances.

Measure the resistances of the given coils.

Example.—Enter results thus:

R	*AS*	*SB*	R_1
1.	49.2	50.8	1.03
1.3	45.5	54.5	1.55
2.3	48.7	51.3	2.42
4.2	46.2	53.8	4.89

Blank to be filled in by student.

R	*AS*	*SB*	R_1

59. TO MEASURE ELECTRICAL RESISTANCES BY MEANS OF A DIFFERENTIAL GALVANOMETER.

References.—As in last Experiment, and, in addition, S. Thompson, pp. 207 and 413.

Apparatus Required.—A differential galvanometer; a battery; a standard resistance-box divided to tenths; resistances to be measured.

Theory of Experiment.—A differential galvanometer is one provided with two coils exactly similar in construction placed symmetrically with reference to the magnetic needle, so that the needle will be uninfluenced by equal currents flowing in opposite directions through the coils.

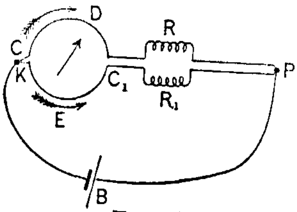

FIG. 38.

Let CDC_1 and CEC_1 represent the two coils of equal resistance, symmetrically placed with regard to the needle, R and R_1 two resistances in series with the coils CDC_1 and CEC_1 respectively.

If K and P be connected through a battery B, the current will flow through the coils in the directions indicated by the arrows.

Since $CDC_1 = CEC_1$, and equal currents through them will produce no deflection, then no deflection will be obtained when $R = R_1$.

If R be a standard resistance, R_1 is directly measured.

Practical Directions.—The following figure shows the connections for one type of instrument, C, Z, P, Q, P_1, Q_1 being terminals to which connections are made. C_1, the middle point of the coils, is connected to one pole of the battery, the other ends of the coils being connected to P and P_1 and through them in series with R and R_1 respectively.

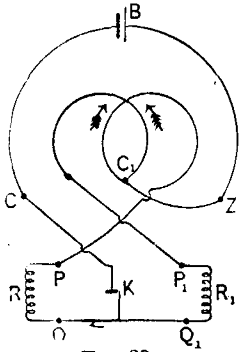

FIG. 39.

The other pole of the battery is connected through C and the contact K to the middle point of R and R_1.

When K is closed the current divides, going through R and R_1 and the coils, as indicated by the arrows.

To test the coils of the galvanometer, adjust R, the stand-

ard, until no deflection is obtained. Then interchange R and R_1 and close the circuit. If now no deflection be obtained, it may be assumed that the coils are equal and accurately placed.

To measure resistances to the second place of decimals, it is necessary to interpolate by taking deflections on either side of the zero for differences of one tenth in R.

Example.—Enter results thus:

R	Deflections.	R_1
100	+ 2	100.04
100.1	− 3	
175.3	− 3	175.25
175.2	+ 3	
75.4	− 3	75.40
75.5	+ 2	

Blank to be filled in by student.

R	Deflections.	R_1

60. TO PROVE THAT THE RESISTANCE OF A WIRE IS DIRECTLY AS ITS LENGTH, AND INVERSELY AS THE CROSS-SECTION; AND TO FIND THE SPECIFIC RESISTANCE OF A WIRE.

References.—Anthony and Brackett, p. 319; Carhart, pt. II. p. 275; Knott, pt. II. p. 189; S. Thompson, p. 402; Barker, p. 700; Ames, p. 333; Nichols and Franklin, vol. II. p. 49; Watson, p. 689.

Apparatus Required.—A B. A. Bridge; a sensitive galvanometer; a standard resistance-box; a battery; a contact-key.

Theory of Experiment.—If l and l_1 be two lengths of wire of the same material but of different diameters, then the resistance of the first is given by the equation

$$R = \frac{4\rho l}{\pi d^2},$$

that of the second by $$R_1 = \frac{4\rho l_1}{\pi d_1^2},$$

assuming that the resistance is directly as the length and inversely as the cross-section, ρ being the specific resistance, d and d_1 the diameters.

Hence $$\frac{R}{R_1} = \frac{d_1^2 l}{d^2 l_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If now the ratio R/R_1 be measured directly by means of the B. A. bridge, equation (1) can be verified.

Having verified the relation, since

$$R = \frac{4\rho l}{\pi d^2},$$

we have

$$\rho = R\frac{d^2\pi}{4l}.$$

Measuring R directly by means of the standard resistance and B. A. bridge, we calculate ρ.

Practical Directions.—Take a meter or so of German-silver wire about No. 18 and, with a draw-plate, draw part of it down to about No. 30.

Coil the two parts together and anneal them thoroughly in a gas-flame, to bring them to the same specific resistance.

Solder the ends of the wires to short thick copper connectors.

The wire with the smaller diameter should be made shorter than the other, so as to make their resistances nearly equal.

Measure carefully by means of a meter scale the lengths of the wires, and the diameters by means of a screw-gauge.

Calculate the ratio, R/R_1, by means of equation (1).

Now connect the two wires in the arms of the B. A. bridge, and adjust until no deflection is obtained on making contact with the sliding contact of the bridge. The ratio of R to R_1 is obtained directly from the ratio of the two lengths, α and α_1, of the bridge wire, or $R/R_1 = \alpha/\alpha_1$.

Remove one of the wires from the bridge, and in its place put the standard resistance.

Measure the resistance of the other wire by the ordinary B. A. bridge method.

The standard resistance should be divided in tenths, so

that a resistance approximately the same as the wire can be unplugged from it.

This observation gives the value R.

Calculate ρ.

Repeat the observation for the second piece of wire and calculate ρ again.

Example.—Enter results thus:

To verify Law. *To calculate ρ.*

l	l_1	d	d_1	R	a	a_1	ρ
100	232	.061	.092	50	47.2	52.8	.000016
a	a_1	$\frac{d^2 l_1}{d_1^2 l}$	a/a_1				
50.3	49.7	1.02	1.01	50	47.5	52.5	.0000158

Blank to be filled in by student.

l	l_1	d	d_1	R	a	a_1	ρ
a	a_1	$\frac{d^2 l_1}{d_1^2 l}$	a/a_1				

61. TO MEASURE THE RESISTANCE OF A GALVANOMETER BY SHUNTING WITH A KNOWN RESISTANCE.

References.—Ames, p. 335; Watson, p. 693; Hastings and Beach, p. 429; S. Thompsom, p. 409; Barker, p. 705; Nichols and Franklin, vol. III. p. 56; Anthony and Brackett, p. 361; Carhart, pt. II. p. 276; Knott, pt. II. p. 190.

Apparatus Required.—A galvanometer; a coil for shunting; a resistance-box; a battery of constant E.M.F.; a reversing-key

Theory of Experiment.—If a resistance R, a galvanometer of resistance G, and a battery with an E.M.F. E and negligible resistance, be connected in series, then the current C is given by the equation

$$C = \frac{E}{R + G}.$$

If θ be the deflection of the galvanometer, then

$$C = K \tan \theta,$$

or $$= K \sin \theta,$$

or $$= K\delta,$$

according as the galvanometer is a tangent, sine, or reflecting instrument, in the latter case δ being the scale deflection.

Hence, taking the second case,

$$\frac{E}{R + G} = K \sin \theta. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If now the galvanometer be shunted by means of a coil S, the other conditions remaining the same, by the theory of

shunts the total current in the circuit is given by the equation

$$C_1 = \frac{E}{R + \frac{GS}{G + S}},$$

and the part of the current through the galvanometer by the equation

$$C_2 = \frac{S}{G + S} C_1.$$

Hence $$C_2 = \frac{E}{R + \frac{GS}{G + S}} \times \frac{S}{G + S}, \quad . \quad . \quad . \quad (2)$$

or $$C_2 = \frac{ES}{R(G + S) + GS}. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

But C_2 is also equal to $K \sin \theta_1$, θ_1 being the deflection of the galvanometer in this case.

hence $$\frac{ES}{R(G + S) + GS} = K \sin \theta_1. \quad . \quad . \quad . \quad (4)$$

Dividing (1) by (4), we have

$$\frac{R(G + S) + GS}{S(R + G)} = \frac{\sin \theta}{\sin \theta_1}. \quad . \quad . \quad . \quad (5)$$

Denoting the ratio $\frac{\sin \theta}{\sin \theta_1}$ by r, and solving for G, we obtain

$$G = \frac{RS(r - 1)}{R + S(1 - r)}, \quad . \quad . \quad . \quad . \quad . \quad (6)$$

from which G can be calculated.

If the galvanometer be a tangent instrument, substitute for r

$$\frac{\tan \theta}{\tan \theta_1}.$$

If the galvanometer be a sensitive reflecting galvanometer, equation (1) becomes

$$K\delta = \frac{E}{R + G}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and (2) becomes

$$K\delta_1 = \frac{E}{R + \frac{GS}{G + S}} \times \frac{S}{G + S} \quad . \quad . \quad . \quad (8)$$

In the case of a reflecting galvanometer, however, R is generally so large as compared with G that

$$R + \frac{GS}{G + S} = R + G,$$

to a close approximation.

In this case, therefore, dividing (7) by 8 we obtain

$$\frac{\delta}{\delta_1} = \frac{G + S}{S},$$

or

$$G = \frac{(\delta - \delta_1)S}{\delta_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

It is usually more convenient to shunt the galvanometer for both observations.

In this case equation (7) becomes

$$K\delta = \frac{E}{R + \frac{G + S}{GS}} \times \frac{S}{S + G}; \quad . \quad . \quad (10)$$

equation (8) become

$$K\delta_1 = \frac{E}{R + \frac{GS_1}{G + S_1}} \times \frac{S_1}{G + S_1}. \quad . \quad . \quad (11)$$

Assuming now that

$$R + \frac{GS}{G + S} = R + \frac{GS_1}{G + S_1},$$

we obtain
$$\frac{S(G + S_1)}{S_1(G + S)} = \frac{\delta}{\delta_1}.$$

Denoting $\frac{\delta}{\delta_1}$ by r we obtain on solving for G

$$G = \frac{(r - 1)SS_1}{S - rS_1}. \quad . \quad . \quad . \quad . \quad . \qquad (12)$$

Practical Directions.—(1) *For Sine or Tangent Galvanometer.*—Connect in series the battery, the resistance-box, and the galvanometer, putting in the reversing-key. A small storage-battery is most suitable, since it has a steady E.M.F. and practically no resistance.

Connect the shunt coil to the terminals of the galvanometer, putting a contact-key in the circuit (see Fig. 40).

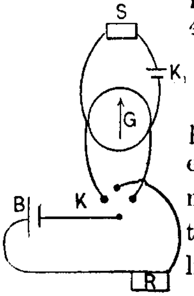

FIG. 40.

Place the galvanometer in the meridian. If a sine or tangent galvanometer with suspended needle be used, care must be taken to eliminate torsion. To do this, lay down the meridian by means of a compass-box and turn the torsion-head of the needle until the needle lies in the meridian.

Now unplug a large resistance from the box R, and close the circuit by means of the reversing-key K, leaving the shunt circuit open.

Adjust the resistance R until a suitable deflection is obtained, about 35°, before turning the galvanometer.

Turn the galvanometer until the needle again reads zero,

that is, occupies the same relative position to the coil as before closing the circuit.

Read the angle through which the galvanometer was turned. If the galvanometer be not provided with a scale for reading off directly the angle through which it was turned, then, after bringing the needle to zero with the current on, open the key and read the angle through which it swings back. This will be the deflection for the sine method.

If a tangent galvanometer be used, adjust the resistance R until a deflection of above 60° is obtained. Reverse the current and repeat the observation, denoting the mean reading by θ.

Now close the shunt circuit simultaneously with the battery. Repeat the observations as before, denoting the mean reading by θ_1.

Find from the tables $\sin\theta$ and $\sin\theta_1$.

Substitute the value $r\left(\dfrac{\sin\theta}{\sin\theta_1}\right)$ with the values for R and S in equation (6), and calculate G.

Repeat the observations three times.

(2) *For a Reflecting Galvanometer.*—It will be necessary to carefully adjust the galvanometer in this case if this be not already done.

First carefully level the instrument by means of the levelling-screws attached.

Adjust carefully the height of the needle by means of the suspension-head until the centre of the mirror is approximately at the centre of the aperture in the coil through which the light is admitted.

If the needle be properly suspended, it will now swing freely. A little further adjustment of the levelling-screws may be necessary.

Place the lamp and scale in front of the mirror, at a distance of about a meter, and adjust the position of the scale until the line joining its centre with centre of the coils is at right angles to the plane of the coils, and its plane parallel to the plane of the coils.

By means of the control magnet adjust the position of the mirror and needle until the light is reflected from the mirror towards the scale.

The position of the reflected light can be determined by holding a sheet of white paper in front of the mirror. The height of the scale can then be adjusted until it receives the reflected image.

Vary the distance between the galvanometer and scale until a clear image is obtained. If the centres of the mirror and scale be in a line at right angles to the plane of the coils, the plane of the scale will be a parallel to the plane of the coils when the ends of the scale are equidistant from the suspension-head.

Adjust the control magnet until the spot of light is at the zero of the scale.

The galvanometer is now ready to be used. Connect the battery, the reversing-switch and a large resistance, R, in series as before. Unplug a large resistance from the box; half a megohm will not be too much to begin with. Close the circuit and observe the deflection. If the deflection be small, reduce the resistance in the circuit until about 300 scale division is obtained. If the spot goes off the scale, increase the resistance until a deflection of about 300 scale divisions is obtained.

Reverse the current and take the mean reading as δ.

Now shunt the galvanometer with a known resistance.

Adjust the shunt, S, until a deflection of about 150 is obtained.

Reverse and read again.

The mean gives δ_1.

Calculate G by formula (9).

Shunt the galvanometer again, S_1, and adjust till a deflection of about 100 scale-divisions is obtained.

Reverse and read again, taking the mean reading.

By means of this reading and δ_1 above calculate G from formula (12).

Example.—Enter results thus:

Sine or Tangent Galvanometer.				Reflecting Galvanometer.			
R	S	Mean Deflection.	G	R	S	Mean Deflection.	G
650	100	28.6					
650	0	65.2	104.5	1×10^6	10000	181.0	
1027	0	60.3					5200
1027	100	26.4	104.7	"	5000	134.5	
825	0	63.8					5100
825	100	27.6	105.0	"	3000	100.5	
Mean G............ ...			104.8	Mean G			5150

Blank to be filled in by student.

Sine or Tangent Galvanometer.				Reflecting Galvanometer.			
R	S	Mean Deflection	G	R	S	Mean Deflection.	G
Mean G...........				Mean G...............			

62. TO MEASURE RESISTANCES BY MEANS OF A WHEATSTONE'S BRIDGE.

References.—As in Experiment 53.

Apparatus Required.—A Wheatstone's bridge; resistances to be measured; a sensitive galvanometer; several batteries; two contact-keys.

Theory of Experiment.—(1) *To Measure the Resistance in a Coil of Wire.*—The theory of the Wheatstone's bridge is exactly the same as that of the B. A. bridge, which is only a simple form of Wheatstone's bridge suitable for measuring small resistances.

In the Wheatstone's bridge the arrangement is as in Fig. 41.

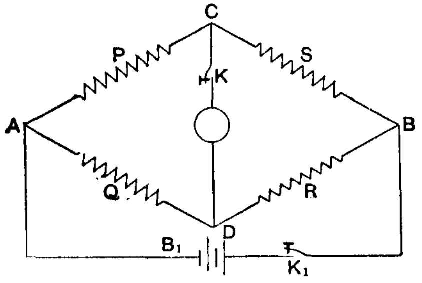

FIG. 41.

In the figure P and Q are fixed resistances, R an adjustable resistance, and S an unknown resistance.

When R is adjusted so that no deflection of the galvanometer is obtained on closing K_1 and K,

$$\frac{P}{Q} = \frac{S}{R},$$

or

$$S = \frac{P}{Q} R.$$

(2) *To Measure the Resistance of the Galvanometer:*

Thompson's Method.—Since no current flows through the galvanometer, when the proper adjustments for the measurement of S have been made, that is, when C and D are at the same potential, the current flowing through each arm of the bridge remains unchanged whether K be closed or not.

Hence, if instead of S a galvanometer were in the arm CB, its deflection due to the passage of the current through the bridge would remain unchanged whether K be closed or not.

It follows, therefore, that if the galvanometer be put in the arm CB, and R be adjusted, until on closing CD directly through K no change of deflection in the galvanometer is observed, C and D have the same potential. For if a current flowed from C to D, the current flowing through the galvanometer would change, causing a change of deflection. Hence

$$\frac{P}{Q} = \frac{G}{R},$$

or

$$G = \frac{P}{Q}R.$$

(3) *To Measure the Resistance of the Battery: Mance's Method.*—If a battery be placed in the arm CB, and the galvanometer again between C and D, on closing K a deflection of the galvanometer will be produced, due to the current from this battery flowing through the system.

If, therefore, R be adjusted until no change in this deflection is observed on closing K_1, C and D are at the same potential as far as the battery between A and B is concerned. Hence

$$\frac{P}{Q} = \frac{B}{R},$$

or

$$B = \frac{P}{Q}R,$$

where B is the resistance of the battery in the arm CB.

Further, since no current flows through the galvanometer from the battery B_1 when the proper adjustments are made, it may be removed altogether and AB connected directly through K_1. R can then be adjusted until one losing K_1 no change in the galvanometer deflection is observed.

In practice it is convenient to bring the galvanometer needle back to zero by means of control magnets.

Practical Directions.—(1) In the ordinary Wheatstone's bridge the arrangement is as in Fig. 42, the letters having

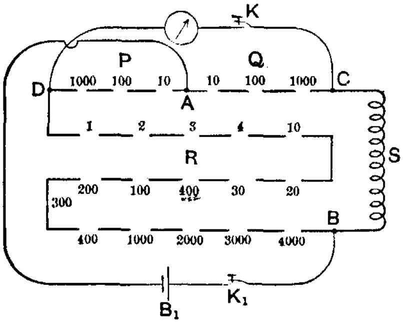

FIG. 42.

the same meaning as in Fig. 41, the numbers indicating the resistance that can be unplugged from the bridge at the points corresponding to the open spaces.

A sensitive galvanometer is required if accurate measurements are to be made. A reflecting galvanometer with telescope and scale or lamp and scale is most suitable.

Complete the connections as in Fig. 42, putting contact-keys in both the galvanometer and battery circuits. Unplug from both P and Q 100 ohms. Shunt the galvanometer with a small resistance while the trial observations are being made.

Unplug a resistance from the arm R, and close the battery key and galvanometer key in the order named.

Observe the direction of the deflection of the galvanometer.

Change the resistance in R until the deflection is in the opposite direction.

The value of S lies between these two values of R.

Continue to adjust R until by changing it 1 ohm the direction of the reflection changes from left to right.

Now open the shunt of the galvanometer so as to increase its sensitiveness.

Change P and Q to 10 and 100 respectively.

Adjust R as before, starting with 10 times the resistance of the smallest of the two previous adjustments.

This will give the resistance to the first decimal place.

Make P 10 and Q 1000, repeating the adjustments for R.

This gives S to two decimal places.

Repeat the observations for the other resistances.

(2) Put the galvanometer in the place of the resistance S.

Put a large resistance in series with the battery between the points A and B, thus diminishing the current flowing through the system.

Adjust this resistance until on closing the battery circuit the deflection of the galvanometer is on the scale.

Repeat the adjustments for P and Q as in (1), leaving the galvanometer circuit closed.

Adjust R until on closing CD directly through a key no change in the galvanometer deflection is observed.

Calculate G.

On changing the values of P and Q in this case, a change in the galvanometer deflections will also take place, and the resistance in series with the battery may have to be adjusted to bring the spot of light again on the scale.

(3) Now put the battery in place of S, and the galvanometer again between C and D.

Put a contact-key in the circuit between A and B.

By means of magnets bring the spot of light to the zero of the scale.

Adjust R until with P equal to 10 and Q equal to 100 no deflection is obtained on closing the key between A and B.

The resistance of the battery is thus obtained to one decimal place.

Repeat the observations for the other batteries given.

Example.—Enter results thus:

Resistance.	P	Q	R	S
Coil A..................	10	1000	2575	25.75
" B..................	10	1000	3645	36.45
" C..................	100	1000	2894	298.4
Galvanometer..........	100	1000	273	27.3
Leclanché battery......	10	100	14	1.40
Daniell battery.........	10	100	85	8.5
Dry cell	10	100	503	50.3

Blank to be filled in by student.

Resistance.	P	Q	R	S

63. (1) TO VERIFY JOULE'S LAW, $JH = C^2Rt$. (2) TO FIND THE VALUE OF J.

References.—S. Thompson, p. 436; Knott, pt. II. p. 196; Barker, p. 709; Hastings and Beach, p. 460; Ames, p. 229; Nichols and Franklin, vol. II. p. 48; Watson, p. 702; Anthony and Brackett, p. 319.

Apparatus Required.—Two copper voltameters or two gas voltameters; two calorimeters with resistance-coils and stirrers; two thermometers.

Theory of Experiment.—(1) Joule's Law may be stated as follows: The heat produced in a given time in any part of a circuit is proportional to the square of the current and to the resistance of that particular part of the circuit.

The law may also be stated thus: If a current C flow through a resistance R for t'' seconds, the work done in driving the current through the resistance is given by the equation

$$W = C^2Rt''. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If now the heat developed, which measures the work done, be utilized to warm a mass of water M from a temperature t to a temperature t_1, we obtain the relation

$$W = JM(t_1 - t), \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where J is the mechanical equivalent of heat, that is, the work done in raising one gram of water through 1° C.

Hence $$JM(t_1 - t) = C^2Rt'',$$

or denoting $M(t_1 - t)$ by H,

$$JH = C^2Rt'' . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

(2) Since $\qquad JH = C^2Rt''$,

or

$$J = \frac{C^2Rt''}{H} . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

From equation (4) J can be calculated if the observations for C, R, t'', and H be made.

In the above equations all the quantities are expressed in C.G.S. units. In making the observations, however, C and R are usually measured in practical units.

Since 1 ampere $= \frac{1}{10}$ C.G.S. units

and 1 ohm $= 10^9$ C.G.S. units,

we have

$$JH = \frac{C^2R10^9t''}{10^2H},$$

or

$$\frac{J}{10^7} = \frac{C^2Rt''}{H}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

the measurements being made in practical units.

The current can be measured by a copper voltameter (see Experiment 52) by means of equation $C = \frac{\delta}{et''}$, where δ is the copper deposited, e the electro-chemical equivalent, and t'' the time.

R can be measured by means of a B.A. bridge. The heat can be measured by means of a water calorimeter, the coil of wire being immersed in it while the current flows.

Practical Directions.—*To Verify the Law.*—A convenient method is to use two calorimeters and two voltameters connected in series (see Fig. 43). MN and M_1N_1 are two copper

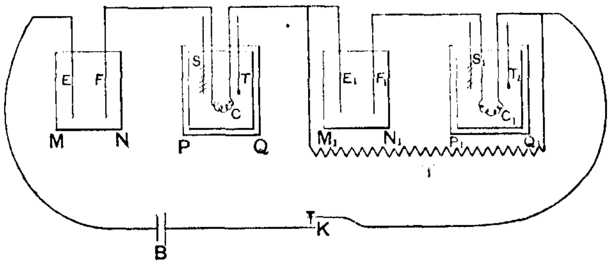

FIG. 43.

voltameters; PQ and P_1Q_1 are two calorimeters with coils C and C_1 of different resistances; r is a resistance shunting one calorimeter and one voltameter in order to obtain a different heating current in each case. If the experiment be performed in this way, two observers are necessary, in which case it will be well for one to take charge of the shunted and the other of the unshunted part of the apparatus. Each observer can make his own observations, checking when possible the observations of the other.

The work required of such observer will then be as follows:

Clean, dry, and weigh the plate of the voltameter on which the copper deposit is to be made (see Experiment 53).

Denote the weight by P.

Weigh carefully the copper vessel of the calorimeter, denoting the weight by W.

Partially fill the vessel with water and weigh again, denoting the weight by W_1; then

$$M = W_1 - W,$$

where M is the mass of the water.

The "water equivalent" of the calorimeter is found by multiplying its mass by .095, the specific heat of copper.

The total mass, therefore, to be heated is

$$M + .095\,W.$$

The water should be reduced in temperature to about as far below the temperature of the room as it will be above that temperature after heating, in order to compensate for the loss of heat by radiation. If the temperature of the room be 17° C., it will be possible to cool the water down to about 10° or 12° and then to heat it to 22° or 24°. In order to get the same rise of temperature the coil of the shunted calorimeter should have a greater resistance than the coil of unshunted one, and the resistance of the shunt adjusted to give a suitable current. The shunt can be found by calculation if the resistance of the calorimeter and voltameter be known approximately, or better still by making a few trials and adjusting its resistance until a rise of one degree is obtained in each in approximately the same time.

As a source of current, the lighting circuit, if direct current, can be used. A couple of storage cells also make a suitable current supply.

Before turning on the current, stir the water in the calorimeter and read the temperature t. This must be done simultaneously by the two observers.

Turn on the current, recording accurately the time, and watch the rise of temperature in the calorimeter, stirring every few minutes.

The required rise of temperature will usually be obtained in from ten to twenty minutes.

Observe carefully the time of turning off the current.

Stir quickly reading the temperature when it reaches its highest point. This will not be for some time after the current is turned off, due to the heat in the wire.

Denote the temperature by t_2.

Wash, dry, and weigh carefully the plate of the voltameter on which the copper was deposited.

Denoting the weight by P_1 and the difference by δ,

$$\delta = P_1 - P.$$

Measure the resistance of the coil by a B. A. bridge.

$$C = \frac{\delta}{.00328t''}; \quad C^2R = \left(\frac{\delta^2}{.00328 \times t''}\right)^2 R;$$

$$H = (t_1 - t)(M + .095\,W).$$

Denoting the ratio of the H obtained from the shunted calorimeter to that obtained from the unshunted by r, and the ratio of the $C\ R$ obtained from the shunted voltameter to that obtained from the unshunted by r_1, then, the law being true,

$$r = r_1.$$

(2) By means of the observations of one calorimeter and voltameter, calculate

$$\frac{J}{10^7}.$$

Precautions.—(1) If the lighting circuit be used, be sure and have a lamp in series with the apparatus. The switch in the lamp will then serve for turning on and off the current.

(2) Be sure the negative pole of the current supply is connected to the plate of the voltameter on which the deposit is to be made.

In Fig. 43, E and E_1 are the negative plates.

(3) If the coil be wound on a frame the water equivalent of the part immersed should also be obtained, and for great accuracy that of the thermometer bulb as well. In the experiment recorded below these connections are put in under the heading $M + .095\,W$.

VOLTAMETER OBSERVATIONS.

	P	P_1	δ	t''	C	R	r
Shunted voltameter.....	53.482	53.665	.183	688	.808	43.75	1.278
Unshunted voltameter...	50.235	60.443	.208	688	.920	25.95	

CALORIMETER OBSERVATIONS.

	W	W_1	$M + .095\,W$	t	t_1	H	r_1
Shunted calorimeter....	90.46	329.75	250.75	8.50	26.75	4576	1.288
Unshunted calorimeter..	82.96	271.65	200.05	8.38	26.13	3551	

$$\frac{J}{10^7} = 4.25.$$

Blanks to be filled in by student.

VOLTAMETER OBSERVATIONS.

	P	P_1	δ	t''	C	R	r
Shunted voltameter.....							
Unshunted voltameter...							

CALORIMETER OBSERVATIONS.

	W	W_1	$M + .095\,W$	t	t_1	H	r_1
Shunted calorimeter....							
Unshunted calorimeter..							

$$\frac{J}{10^7} =$$

64. COMPARISON OF RESISTANCES BY CAREY FOSTER'S METHOD.

References.—As in Experiment 57; S. Thompson, p. 420.

Apparatus Required.—A B. A. bridge; a low-resistance galvanometer; the two resistances to be compared; two unknown but nearly equal resistances; a thermometer for taking the temperature of the coils under comparison; a water-bath in which to immerse them; mercury-cup connectors; a battery; a reversing-switch.

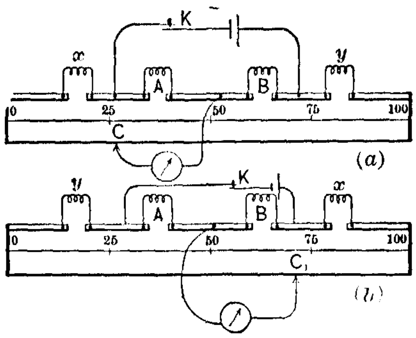

FIG. 44.

Theory of Experiment.—Let the connections first be made as in Fig. 44 (*a*), where x and y are the resistances to be compared, A and B the "ratio resistances," which are nearly equal to one another; C the position of the sliding contact when a balance is obtained on the bridge-wire; a and b the distances of C from the ends of the bridge-wire; l the length and σ the resistance per centimetre of the wire.

Let r and r_1 be the resistances of the intervening copper straps.

Then we have

$$\frac{A}{B}=\frac{x+r+\sigma a}{y+r_1+\sigma b}. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

On interchanging x and y as in Fig. 44 (b), we have on obtaining a balance again, where the new position C_1 is distant a_1 and b_1 from the ends of the bridge.

$$\frac{A}{B} = \frac{y + r + \sigma a_1}{x + r_1 + \sigma b_1}. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence, equating (1) and (2),

$$\frac{x + r + \sigma a}{y + r_1 + \sigma b} = \frac{y + r + \sigma a_1}{x + r_1 + \sigma b_1}. \quad . \quad . \quad . \quad (3)$$

By adding unity to each side of (3) we have

$$\frac{x + r + \sigma a + y + r_1 + \sigma b}{y + r_1 + \sigma b} = \frac{y + r + \sigma a_1 + x + r_1 + \sigma b_1}{x + r_1 + \sigma b_1}, \quad (4)$$

or

$$\frac{x + r + y + r_1 + \sigma(a + b)}{y + r_1 + \sigma b} = \frac{y + r + x + r_1 + \sigma(a_1 + b_1)}{x + r_1 + \sigma b_1}. \quad (5)$$

Hence, since $(a + b) = (a_1 + b_1)$, the numerators of these fractions are equal, and therefore

$$y + r_1 + \sigma b = x + r_1 + \sigma b_1,$$

or

$$y - x = \sigma(b_1 - b), \quad . \quad . \quad . \quad . \quad . \quad (6)$$

a formula quite independent of the resistances A and B, and also of the other resistances, r, r_1, which enter as errors into the ordinary B. A. bridge methods.

It is therefore clear that this is a very convenient and accurate way of comparing small resistances.

To obtain the greatest accuracy by this method, it is essential that the resistances A and B should not differ much from x and y.

In using formula (6) it is of course essential to know σ, the resistance per linear unit of the bridge-wire.

The value of σ may be easily determined. A simple method is to determine the difference between a known standard resistance P and a negligible resistance Q. In which case, from formula (6), we have

$$(P - Q) = \sigma(a_2 - a_3);$$

or, since $Q = 0$, we have

$$\sigma = \frac{P}{a_2 - a_3}, \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where a_2, a_3 are again the distances of the balance-points from one end of the bridge.

A simpler but less accurate method is to measure the resistance of the whole bridge-wire l by means of another B. A. bridge, and divide the resistance by the length l of the bridge-wire.

Or again suppose that instead of x and y two standard one-ohm coils be put in the bridge and shunted successively with a standard 10-ohm coil and the system balanced in each case. Then, the difference of the resistances in this case being $\frac{1}{11}$ ohms,

$$\tfrac{1}{11} = \sigma(j - j_1),$$

where j and j_1 are the distances from one end of the bridge-wire of the balance-points in the two adjustments.

Hence $$\tfrac{1}{11} = \sigma(j - j_1),$$

or $$\sigma = \frac{1}{11(j - j_1)}.$$

This is a simple and accurate method of measuring σ.

Practical Directions.—*To Compare x and y.*—Make connections as shown in Fig. 44 (a), K being a reversing-key.

The coils should be connected in the bridge by means of mercury cups.

See that the terminals of all the coils are well down on the bottoms of the mercury cups.

Adjust the sliding contact until a balance is obtained.

Reverse the battery current and balance again to eliminate any error due to thermal effects.

Take the mean of the two readings for the position of C.

Interchange x and y, Fig. 44 (b), and proceed as before, obtaining a mean value for the position of C_1.

Then $x - y = \sigma(b_1 - b)$, where $b_1 - b$ is the length from C to C_1.

To Determine σ.—Use a standard resistance somewhat smaller than that of the bridge-wire, and also a thick copper strap, the resistance of which can be neglected, in place of x and y, and repeat the observations as above, reversing the current and taking a mean reading in each case.

Denote the distances of C and C_1 from the zero end of the bridge by a_2 and a_3.

Either of the other methods mentioned on page 211 will do equally well.

Calculate the value of σ from formula (7), and the value of $x - y$ from (6).

If either x or y be known absolutely, the other will be completely determined.

If one of the resistances be a German-silver standard or other ordinary standard, it will be necessary to take its temperature in a water-bath, and correct for it. The temperature cannot, perhaps, be known closer than $\frac{1}{10}°$ on account of the very thick coating of paraffin and silk around the resistance, but the error due to that discrepancy would be trifling. If the standard be of resistance R and correct at a temperature t, and the observations be made at a temperature

t_1, then the true resistance, R_1, of standard **at** temperature t_1 is given by equation

$$R_1 = R\{1 + \alpha(t_1 - t)\},$$

α being the temperature coefficient of the wire.

All the bridge readings should be taken to $\frac{1}{10}$ of a millimetre.

Use for x a standard one-ohm coil and calculate y.

Precautions.—It is necessary that the terminals of the coils should make a good steady contact with the bridge. They should therefore be well amalgamated, and if necessary tied down to the bottom of the cups with rubber bands.

The battery current must always be reversed at each balance-point, in order to eliminate the effect of thermo-currents.

If a storage battery is used for the source of current, be careful not to short-circuit it.

Example.—Enter results thus:

Observations for σ.				Observations for $x - y$.				
P	Mean a_2	Mean a_1	σ	b	b_1	t	t_1	$\sigma(b_1 - b)$
.1	57.5	39.5	.00555	62.5	39.1	16.4	17.0	.013

Blank to be filled in by student.

Observations for σ.				Observations for $x - y$.				
P	Mean a_2	Mean a_1	σ	b	b_1	t	t_1	$\sigma(b_1 - b)$

65. TO CALIBRATE THE SLIDE-WIRE OF A B. A. BRIDGE BY CAREY FOSTER'S METHOD.

References.—As in Experiment 64.

Apparatus Required.—The B. A. bridge which is to be calibrated; a supplementary bridge-wire having a sliding contact, or a second B. A. bridge; a small resistance having a value equal to about one-tenth the resistance of the whole bridge-wire; a stout copper strap of negligible resistance; two pairs of mercury cups; a reversing-switch; an ordinary contact-key; a battery; a suitable low-resistance galvanometer.

Theory of Experiment.—In the previous experiment it was assumed that the resistance of a unit length of the bridge-wire was uniform throughout.

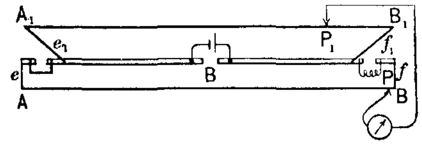

FIG. 45.

The object of the present experiment is to test the uniformity of the wire by determining the resistances of equal portions of the wire at different positions along its length.

Let the two bridge-wires AB and A_1B_1, with the small coil and copper strap, be connected as shown in Fig. 45, the sliding-contact P being as close as possible to B, the other sliding-contact, P_1, being adjusted until no galvanometer deflection is obtained. Then, denoting the resistance of AB by l, of A_1B_1 by l_1, of the small coil (called the "*gauge*") by G, of the copper strap (called the "*connector*") by C,

and the resistances of the connecting wires by e, e_1, f, f_1, as shown in Fig. 45, we have, from the theory of the B. A. bridge, the following relation:

$$\frac{C+e+l}{G+f}=\frac{e_1+r_1}{f_1+l_1-r_1}, \quad . \quad . \quad . \quad (1)$$

r_1 being the resistance of the length A_1P_1 of the wire A_1B_1.

If C and G be now interchanged and P moved along the wire AB until a new balance-point is obtained, we obtain the equation

$$\frac{G+e+r}{C+f+l-r}=\frac{e_1+r_1}{f_1+l_1-r_1}, \quad . \quad . \quad . \quad (2)$$

where r is the resistance of the portion of the wire between A and the new position of P.

Equating (1) and (2), adding unity to each side and inverting, we obtain

$$\frac{G+f}{C+e+l+G+f}=\frac{C+f+l-r}{C+G+f+e+l}.$$

Hence $$G+f=C+f+l-r,$$

or $$G-C=l-r; \quad . \quad . \quad . \quad . \quad . \quad (3)$$

that is, the resistance of the length $l-r$ through which the slider was moved to obtain a balance is equal to the difference in resistance between the connector and gauge.

If now P be left at rest and P_1 moved until a balance is again obtained, it will be found that the length of wire over

which P_1 is moved is equal to the difference between the connector and gauge. Hence both wires are calibrated simultaneously, since lengths on both wires of equal resistance, having a value equal to the difference between the gauge and connector, are obtained.

The resistance of the gauge may be measured as in the last experiment, using the connector as the negligible resistance.

The value of σ for each length is obtained by dividing the value of $G - C$ by the length.

Practical Directions.—Make connections as in Fig. 45, putting a reversing-switch in the battery circuit.

The connector, C, and gauge, G, should be connected in by means of mercury cups, the contact-points being well amalgamated.

With the connector and gauge as figured, set P close to B and balance by moving P_1. Reverse the battery current and balance again.

The mean reading gives the length corresponding to the resistance $l_1 - r_1$, the first calibrated length on $A_1 B_1$ if P be at the extreme end of AB.

Interchange C and G, and, keeping P_1 fixed, move P until a balance is obtained. Reverse the battery current as before and balance again.

The mean reading gives the length corresponding to $l - r$, the first calibrated length on AB.

Again interchange C and G, and move P_1 until a balance is obtained, repeating the adjustments and observation along the wire alternately until the end of one of them is reached.

Determine $G - C$ by one of the methods indicated in the last experiment and calculate the value of σ for each of the lengths $l - r$.

Example.—Enter results thus:

$G - C = .053.$

Reading Bridge AB.	Difference.	σ	Reading Bridge A_1B_1.	Difference.	σ_1
0	....		8.75	....	....
9.56	9.56	.00554	17.44	8.69	.00610
19.16	9.60	.00552	26.04	8.60	.00616
28.73	9.57	.00554	34.79	8.75	.00605
38.36	9.63	.00550	43.46	8.67	.00611
47.96	9.60	.00552	52.19	8.73	.00607
57.51	9.55	.00555	60.83	8.64	.00613
67.09	9.58	.00553	69.48	8.65	.00613
76.63	9.54	.00555	78.18	8.70	.00609
87.27	9.64	.00549	86.88	8.70	.00609
96.77	9.60	.00552	95.53	8.65	.00613
Mean value of σ		.00552	Mean value of σ_1		.00610

Blank to be filled in by student.

Reading Bridge AB.	Difference.	σ	Reading Bridge A_1B_1.	Difference.	σ_1
Mean value of σ			**Mean value of σ_1**		

66. VARIATION OF RESISTANCE WITH TEMPERATURE. TO DETERMINE THE TEMPERATURE COEFFICIENT OF A CONDUCTOR.

References.—S. Thompson, p. 403; Nichols and Franklin, vol. II. p. 50; Knott, pt. II. p. 199; Watson, p. 690; Barker, p. 714; Carhart, pt. II. p. 276; Anthony and Brackett, p. 319; Hastings and Beach, p. 425.

Apparatus Required.—A Wheatstone bridge, preferably one of the dial pattern; a sensitive low-resistance galvanometer; a couple of cells; a hypsometer; a vessel containing ice or snow saturated with water; some mica; a metre of fine-drawn wire (platinum .006 in. is suitable); a reversing-key; a contact-key for making a permanent contact.

Theory of Experiment.—For small changes of temperature the increase of resistance of pure metals is found to be nearly proportional to the increase of temperature. If R_0 be the resistance of a coil of wire at temperature 0 C., and R its resistance at temperature t, then

$$R_t = R_0(1 + \alpha t), \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where α is the temperature coefficient of the wire.

Hence
$$\alpha = \frac{R_t - R_0}{R_0 t}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If t be 100°, the boiling-point of water, (2) becomes

$$\alpha = \frac{R_{100} - R_0}{100 \, . \, R_0}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If R_{100} and R_0 be measured, α can be calculated.

Practical Directions.—Take about a metre of .006 in. platinum wire and anneal it by passing it slowly through a

bunsen flame. If wire other than platinum be used, the method of treatment will depend on the material. Solder to each end of the platinum wire a piece of No. 20 copper wire about $1\frac{1}{2}$ metres in length.

Take a strip of mica about 8 centimetres in length and 1 centimetre wide and notch it with file-cuts about a millimetre apart. On this wind the platinum wire, making a coil as in Fig. 46.

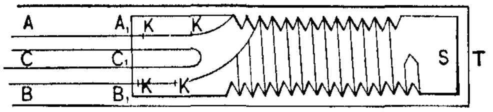

Fig. 46.

The two copper wires, marked AA_1, BB_1, should be fastened to the mica by small loops of wire passing through the mica at points corresponding to those marked K. This should be done before the coil is wound.

Now take a piece of copper wire of the same size and equal in length to the other two and, bending it at its middle point, fasten it to the mica in position corresponding to CC_1.

Insert the coil into a glass tube about 40 centimetres in length and just large enough to receive it.

Fit a cork tightly in the tube, allowing the copper wires to come out by means of notches in the cork.

If the wires be double-covered, they may now be bound together in several places along their length so as to make the whole comparatively rigid, care being taken to mark the ends of the leads connected with the coil.

The coil is now ready for use.

The wires AA_1, BB_1 serve to connect the coil into the bridge, while CC_1 compensates for their resistance when connected into the opposite arm.

Connect the coil and compensating leads as in Fig. 47, M being an additional insulated terminal, by means of which the coil and compensating lead are put into different arms of the bridge, P and Q the ratio arms, R the adjustable arm with the compensating leads C, and S the coil.

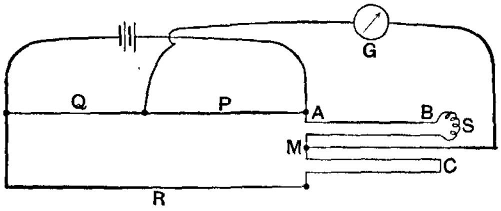

FIG. 47.

Immerse the tube containing the coil to about half its length in the vessel containing the snow or ice. Making P and Q 10 and 10,000 respectively, adjust R until no deflection is obtained, and continue to adjust R until the resistance of the coil becomes steady. This usually takes about 5 minutes. The resistance of a coil of the size mentioned above is about 4 to 5 ohms, so that R will be about 4000 ohms.

Now insert the tube into the hypsometer, keeping its end two or three inches above the water, and let the steam flow around it freely until a steady temperature, as indicated by the resistance, is obtained.

Read the barometer and find the temperature of steam corresponding to the barometric pressure.

Calculate α.

Precautions.—The battery circuit should connect the junction of the two large resistances, Q and R, to the junctions of the two small ones, P and S, as in Fig. 47, in order to keep the current flowing through the system very small; otherwise the coil will become heated by it.

Thermo-electric effects can be eliminated by using a reversing-key.

Repeat the observations several times.

Example.—Enter results thus:

	P	*Q*	*R*	*S*	*B*	*t*	*a*
Ice.............	10	10000	4226.	4.226	75.33	99.75	.00341
Steam..........			5665.	5.665			
Ice.............			4227.	4.227	"	"	.00341
Steam..........			5665.	5.665			
Ice.............			4226.	4.226			.00341
Steam..........			5665.	5.665			

Blank to be filled in by student.

	P	*Q*	*R*	*S*	*B*	*t*	*a*
Ice.............							
Steam..........							
Ice.............							
Steam..........							
Ice.............							
Steam..........							

67. TO MEASURE A VERY SMALL RESISTANCE.

References as in Experiment 56.

Apparatus Required.—A standard .01 ohm; a small resistance to be measured; a storage-battery; a plug contact-key; two Pohl commutators; a sensitive galvanometer.

Theory of Experiment.—The methods described in the previous experiments are not suitable for the measurement of

very large or very small resistances. The present is one of several methods which might be used to determine the resistance of a conductor whose resistance is very small.

Let a standard resistance R and the unknown resistance X be connected in series with a battery B, Fig. 48. Then,

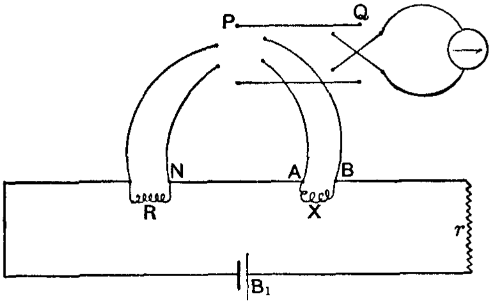

FIG. 48.

by Ohm's law, the potential between the points N and M is to that between A and B in the ratio of the resistance R to X, or

$$\frac{V}{V_1} = \frac{R}{X}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where V and V_1 denote the potential difference between the points M and N and the points A and B respectively.

If now MN and AB be connected successively to terminals of a galvanometer, the deflections of the galvanometer will also be proportional to the potential differences between the points.

Hence, if δ and δ_1 be the deflections,

$$\frac{X}{R} = \frac{\delta_1}{\delta},$$

or

$$X = \frac{\delta_1}{\delta} R. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

It is usually necessary, however, to have an adjustable resistance in the galvanometer circuit which can be adjusted so as to give approximately equal deflections.

If S be the resistance in series with the galvanometer when its terminals are connected to M and N, and S_1 when they are connected to A and B; then

$$K\delta = \frac{V}{S + G},$$

and
$$K\delta_1 = \frac{V_1}{S_1 + G}.$$

Hence
$$\frac{\delta}{\delta_1} = \frac{(S_1 + G)V}{(S + G)V_1},$$

or
$$\frac{V_1}{V} = \frac{\delta_1(S_1 + G)}{\delta(S + G)},$$

or, substituting from (1) and solving for X_1,

$$X_1 = \frac{S_1 + G}{S + G} \times \frac{\delta_1}{\delta}R. \quad . \quad . \quad . \quad . \quad (3)$$

If G be not known, it can be found by the method described in Experiment 61.

Practical Directions.—Connect in series a storage-battery, the standard resistance, the unknown resistance, and an adjustable resistance, r (a metre or so of bare German-silver wire No. 20 is usually suitable).

Remove from one of the Pohl commutators, P, the thick wires on the base connecting the mercury-cups.

Connect (see Fig. 48) the terminals of the two resistances, R and X, to the two pairs of terminals from which the connections have been removed, and the remaining pair of ter-

minals, in which the rocker dips, to the corresponding pair of the second commutator, Q.

Connect the galvanometer terminals to a remaining pair of terminals of the second commutator.

By rocking the first switch AB and MN can be successively connected to the galvanometer, and by rocking the second the current can be reversed in the galvanometer circuit.

A suitable unknown resistance can be made from a piece of thick copper wire stretched between two terminals.

Half a metre of No. 10 wire gives a suitable small resistance.

Rock the switch to which the resistances are attached so that the potential from R is on the galvanometer terminals.

Adjust S until a suitable deflection is obtained.

Reverse the current and take the mean reading.

Repeat the observations for X, taking a mean deflection.

Repeat the whole series of observations several times.

Measure G by method of Experiment 61.

Example.—Enter results thus:

R	S	δ	S_1	δ_1	G	x
.01	10000	250	0	200	2050	.00136
Mean value of x....................................						

Blank to be filled in by student.

R	S	δ	S_1	δ_1	G	x
Mean value of x..						

68. TO MEASURE A VERY LARGE RESISTANCE.

References.—As in Experiment 56.

Apparatus Required.—A sensitive high-resistance galvanometer; a number of cells; a megohm; a resistance-box for shunting the galvanometer; a reversing-switch; resistances to be measured.

Theory of Experiment.—As stated in the last experiment the ordinary bridge methods are not suitable for the measurement of very large resistances. The method here described is a very simple and direct one, similar to that of Experiment 56, only large resistances and a sensitive galvanometer are used.

Let a galvanometer G, a large unknown resistance X, and a battery B be connected in series with the galvanometer. Let the E.M.F. of the battery used be E, the deflection obtained δ, and the current through the galvanometer C.

Then
$$C = \frac{E}{X + B + G} = K\delta. \quad . \quad . \quad . \quad (1)$$

Now let the resistance X be replaced by a known resistance R, giving a deflection δ_1; then

$$C_1 = \frac{E}{R + B + G} = K\delta_1. \quad . \quad . \quad . \quad (2)$$

Hence
$$\frac{R + B + G}{X + B + G} = \frac{\delta}{\delta_1},$$

or, neglecting B,
$$\frac{R + G}{X + G} = \frac{\delta}{\delta_1}. \quad . \quad . \quad . \quad . \quad (3)$$

If, however, X be a very large resistance, say a number of

megohms, and R one megohm, it will be necessary to shunt the galvanometer when R is in the circuit.

If S be the resistance of the shunt, (2) becomes

$$C_1 = \frac{E}{R + B + \frac{GS}{G + S}} \times \frac{S}{G + S} = K\delta_1,$$

or
$$\frac{ES}{(R + B)(G + S) + GS} = K\delta_1. \quad . \quad . \quad . \quad (4)$$

Combining (1) and (4), we obtain

$$\frac{(G + S)(R + B) + GS}{S(X + B + G)} = \frac{\delta}{\delta_1}. \quad . \quad . \quad . \quad (5)$$

Denoting $\frac{\delta}{\delta_1}$ by r, neglecting B, and solving for X, we obtain

$$X = \frac{R(G + S)}{rS} + \frac{G}{r}(1 - r). \quad . \quad . \quad . \quad (6)$$

If the galvanometer be provided with a shunt-box, then the ratio $\frac{G + S}{S}$ is known and is usually 10, 100, or 1000; $\frac{G}{r}(1 - r)$ is generally small as compared with R and may be neglected.

Hence
$$X = \frac{R(G + S)}{rS}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

a simple formula for calculation.

If, however, an adjustable resistance be used to shunt the galvanometer, S can be adjusted until

$$\delta = \delta_1 \quad \text{or} \quad r = 1,$$

and hence equation (6) becomes

$$X = \frac{R(G + S)}{S}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

also a simple formula for calculation.

Practical Directions.—A very sensitive galvanometer is necessary for this experiment.

(1) Measure the resistance of a deep line drawn by a lead-pencil on a strip of white paper.

(2) Measure the insulation of a coil of cotton-covered twin-wire.

(3) Measure the insulation of a coil of rubber-covered wire.

In the first case the strip of paper can be connected into the circuit by means of terminals screwed into a piece of board, the paper being stretched on the surface of the board. Connect in a sufficient number of batteries to obtain a deflection of about 200 scale-divisions.

The reversing-switch should be in the circuit so that the current can be reversed and the mean reading taken.

Substitute a megohm for the unknown resistance, and shunt the galvanometer to obtain a suitable deflection or the same deflection, according as (7) or (8) is to be used in the calculation.

In the second case, separate the twin-wires at one end and connect the two wires at the other end in series with the galvanometer and battery.

Repeat the observations and calculations as above.

In the third case, place the coil of wire in a metal vessel

containing enough water to just cover it, keeping about one foot of the covered wire at each end above the surface of the water.

Connect in series with the galvanometer and battery one end of the wire and the edge of the metal dish.

The resistance thus measured is the resistance of the whole insulations.

If the rubber-covered wire be soaked in water several hours before the trial, so much the better.

In both the second and third cases the galvanometer should be shunted with a large shunt for trial observations to avoid sending large currents through it in case of a very faulty insulation.

The wires connecting the galvanometer to the unknown resistance should in each case be carefully insulated from the other wires of the circuit, and the current closed by means of a tapping contact-key.

Example.—Enter results thus:

x	R	G	S	$\frac{G+S}{S}$	δ	Resistance of x.
Pencil mark....	1Ω	5500	110	51	200	51Ω
Twin-wire.....	"	"	180	31	330	31 "
Rubber-covered wire.........	"	"	—	—	—	Infinity

Blank to be filled in by student.

x	R	G	S	$\frac{G+S}{S}$	δ	Resistance of x.
Pencil mark....						
Twin-wire.....						
Rubber-covered wire.........						

69. COMPARISON OF ELECTROMOTIVE FORCES OF BATTERIES, BY TANGENT GALVANOMETER.

References.—Anthony and Brackett, pp. 317, 334–340, and 359; Knott, pt. II. pp. 159–166 and 185; Barker, pp. 561, 699, and 758, 759; Nichols and Franklin, vol. II. pp. 54 and 79–85; Hastings and Beach, pp. 390–395; S. Thompson, pp. 154 and 163–174; Carhart, pt. II. pp. 233–253 and 273; Ames, pp. 233, 306, and 310–315; Watson, pp. 674, 688, and 815–823.

Apparatus Required.—A sine or tangent galvanometer; a resistance-box; a contact-key; batteries to be compared.

Theory of Experiment.—If a current from a battery flow through a resistance R, a galvanometer of resistance G, then, if a tangent galvanometer be used,

$$C = \frac{E}{B + G + R} = K \tan \theta,$$

B being the resistance and E the E.M.F. of the battery, and θ the deflection of the galvanometer.

Hence $$E = K(B + G + R) \tan \theta. \quad . \quad . \quad . \quad (1)$$

If now another battery be used, E.M.F. E_1, resistance B_1, and an external resistance R_1, producing a deflection θ_1, then

$$E_1 = K(B_1 + G + R_1) \tan \theta_1. \quad . \quad . \quad . \quad (2)$$

Hence $$\frac{E}{E_1} = \frac{(B + G + R) \tan \theta}{(B_1 + G_1 + R_1) \tan \theta_1}; \quad . \quad . \quad . \quad (3)$$

or if a sine galvanometer be used,

$$\frac{E}{E_1} = \frac{(B + G + R) \sin \theta}{(B_1 + G + R_1) \sin \theta_1}.$$

If the resistance of the batteries and the galvanometer be known, the ratio, E/E_1, can be found at once.

If the battery and galvanometer resistance be not known, they must first be calculated by the method used in measuring the resistance of a galvanometer and battery in Experiment 56.

Practical Directions.—Connect a Daniell cell in series with a resistance-box, the galvanometer and a reversing-key.

Unplug from the box a large resistance.

Close the circuit by means of the contact-key.

Adjust the resistance R till a suitable deflection is obtained, say about 60°.

Read the deflection, δ.

Reverse the current and read the deflection again, δ_1.

$$\theta = \frac{\delta + \delta_1}{2}.$$

Again adjust the resistance until the deflection obtained be about 30°, denoting the new resistance by R_1.

Reverse the current and read as before, δ_2, δ_3.

$$\theta_1 = \frac{\delta_2 + \delta_3}{2}.$$

Then
$$\frac{B + G + R_1}{B + G + R} = \frac{\tan \theta}{\tan \theta_1},$$

assuming that a tangent galvanometer is used.

From this equation calculate the resistance $B + G$.

Make similar observations with another battery, and calculate the resistance of $B_1 + G$ from equation

$$\frac{B_1 + G + R_2}{B_1 + G + R_3} = \frac{\tan \theta_3}{\tan \theta_2},$$

B_1 being a second battery, θ_2 and θ_3 the mean deflections.

These observations will also give the ratio E/E_1 by taking one observation in each case, e.g.,

$$\frac{E}{E_1} = \frac{(B + G + R)\tan\theta}{(B_1 + G + R_2)\tan\theta_2}.$$

Substitute for $B + G$ and $B_1 + G$ the values found, and for R and R_2 the resistances unplugged from the box.

Assume the electromotive force of the Daniell cell to be 1.08, and calculate that of the other.

Compare with the Daniell cell, a Leclanché, a dry battery, a storage battery.

Example.—Enter results thus:

Battery.	R	R_1	θ	θ_1	$B+G$	E
Daniell....................	20	40	50.66	33.56	3.8	1.08
Leclanché..................	20	40	60.20	42.33	1.9	1.42
Dry battery................	20	40	42.45	31.05	20.3	1.35
Storage-cell...............	30	60	63.00	44.85	.8	2.25

Blank to be filled in by student.

Battery.	R	R_1	θ	θ_1	$B+G$	E

70. COMPARISON OF THE E.M.F. OF BATTERIES BY POTENTIOMETER METHOD.

References.—As in last experiment and, in addition, S. Thompson, p. 421.

Apparatus Required.—A sensitive galvanometer; a potentiometer; a three-way plug-key; a storage battery; a battery of constant E.M.F.; batteries for comparison.

Theory of Experiment.—Suppose E a battery of constant E.M.F., the poles of which are connected to a wire AB of resistance R, A being the negative pole.

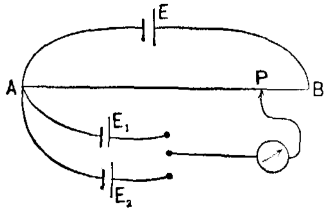

Fig. 49.

Let the resistance of the battery and connection be denoted by r; then

$$C = \frac{E}{R + r}.$$

If the negative pole of any other battery of E.M.F. E_1 be connected to A and through a galvanometer to a point P on AB such that no current flow through the galvanometer, then the E.M.F. of E_1 must be equal to the difference of potential between A and P produced by the battery E.

Hence $$C = \frac{E_1}{R_1}$$

if R_1 be the resistance of AP.

Hence $$\frac{E_1}{R_1} = \frac{E}{R + r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If r be negligible as compared with R, or of known value, the ratio E to E_1 can thus be determined.

If another battery, E_2, be similarly connected, and a point P_2 be found such that no current passes through the galvanometer, then

$$\frac{E_2}{R_2} = \frac{E}{R + r}.$$

Hence $$\frac{E_1}{E_2} = \frac{R_1}{R_2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

a comparison of E_1 and E_2 independent of r.

Practical Directions.—Connect to the ends of a potentiometer-wire the poles of a storage battery E, Fig. 49, with negative pole at A.

Connect the negative pole of a battery, E_1, with which the others are to be compared, to A, and through a three-way plug key to a galvanometer which is connected to the sliding-contact on the potentiometer.

Connect to A and the galvanometer another battery, E_2, through the third connection of the three-way key.

Adjust the sliding-contact so that when E_1 is connected through the galvanometer no deflection is obtained.

Read the distance AP on the scale attached to the potentiometer.

Connect E_2 with the galvanometer, and adjust again for no deflection.

Read the length AP_2 as before.

Denote these lengths by l_1 and l_2.

Now, since the wire of the potentiometer is uniform,

$$\frac{E_1}{E_2} = \frac{R_1}{R_2} = \frac{l_1}{l_2}.$$

Hence $$E_2 = \frac{l_2}{l_1} \times E_1.$$

Now replace E_2 by another battery E_3, and compare as before.

$$E_3 = \frac{l_3}{l_1} \times E_1.$$

For each comparison the length l_1 should be verified.

Compare with a Daniell cell (E_1) as standard the cells given. A Clark cell is preferable to a Daniell if the apparatus be suitable. In this case care must be taken not to short-circuit the Clark cell.

Example.—Enter results thus:

Battery.	l	l/388.0	E
Clark cell..........	388.0	.....	1.434
Daniell.............	292.2	.753	1.08
Leclanché	378.8	.976	1.40
Bunsen.............	513.5	1.323	1.90
Bichromate.........	554.7	1.430	2.05
Grove....	514.1	1.325	1.90
Dry battery........	351.8	.906	1.30

Blank to be filled in by student.

Battery.	l	l/l_1	E

71. TO CALIBRATE AN AMMETER, BY MEANS OF A GAS-VOLTAMETER.

References.—S. Thompson, p. 209; Nichols and Franklin, vol. II. p. 89; Hastings and Beach, p. 421.

Apparatus Required.—A gas-voltameter; a couple of storage-cells or other suitable source of E.M.F.; a rheostat; a reversing-key; a stop-watch; the galvanometer or ammeter which is to be calibrated.

Theory of Experiment.—If the ammeter or galvanometer to be calibrated be connected in series with any standardizing instrument, the indications of the latter being proportional at any instant to the current passing through it, the indications of the first instrument may be reduced to their value in current, or, in other words, the instrument may be calibrated.

The present experiment is one of relative calibration only. For this purpose a convenient standardizing instrument is a form of gas-voltameter devised by Prof. Ayrton. In this voltameter the electrolytic chamber is sealed up, and the rate at which the mixed gases, hydrogen and oxygen, are given off is observed by the rate of rise of the electrolyte in a tube whose lower end reaches to the bottom of the electrolytic chamber. The tube is graduated above the voltameter, and the time required for the liquid to rise through a given number of divisions is inversely proportional to the current passing through the voltameter.

Therefore, for any given current through the voltameter and ammeter in series, the reciprocal of the time taken to rise through one division is a measure of the current producing the corresponding deflection of the ammeter.

A curve can therefore be plotted co-ordinating the re-

ciprocals of these times and their corresponding ammeter deflections. This curve is a relative calibration curve for the ammeter.

If an absolute calibration were required, it would be necessary to determine the quantity of gas deposited in a given time and to calculate the current corresponding to each deflection.

Practical Directions.—Connect the gas-voltameter, the source of E.M.F., and the rheostat or other adjustable resistance in series through a reversing-key to the ammeter, so that the current may be reversed through the ammeter without reversing it through the voltameter.

Start with a large current through the instrument sufficient to give 60 or 70 deflections.

If possible, read both ends of the needle, δ, δ_1.

Observe with the stop-watch the time required for the electrolyte to rise from the bottom graduation on the tube to the top one. If the instrument be one in which deflections can be read on both sides of the zero, reverse the current and as before read both ends of the needle, δ_2, δ_3, and the time of rise of the electrolyte.

Repeat the observation for a number of different deflections.

This can be done by varying the resistance in the circuit.

With the smaller currents it will not be necessary to wait until the liquid has risen through the whole length of the tube, but can be taken through a few of the graduations and the time it would take for the whole calculated.

Plot a curve with galvanometer deflections (θ) for abscissas, and reciprocals of times $\frac{1}{(t)}$ for ordinates.

Precautions.—If the ammeter has a single turn or only a few turns in its coil, it will be necessary to connect it so that

the rest of the apparatus is at least a meter distant from it. Otherwise the current in that part of the circuit will affect the ammeter readings.

Do not short-circuit the storage-battery.

Example.—Enter results thus:

δ	δ_1	δ_2	δ_3	θ	t''	$\frac{1}{t}$
6.5	7.5	7.5	8.5	7.5	234	.00431
9	10	10	10.5	10.1	155	.0064
11	12.5	10	11	11.1	131	.0076
16	17	17	18.5	17.1	84	.0119
26	25.5	24	24.5	25	45	.0222
39	39.5	42	42.5	40.7	25	.04
53	54	50.5	51.5	52.2	15	.066
62	61	64	65	62.7	8.5	.116

Blank to be filled in by student.

δ	δ_1	δ_2	δ_3	θ	t''	$\frac{1}{t}$

Show curve.

72. TO DETERMINE THE CONSTANT OF A SIEMENS ELECTRO-DYNAMOMETER.

References.—S. Thompson, p. 392; Hastings and Beach, p. 423; Nichols and Franklin, vol. II. p. 211; Anthony and Brackett, p. 358; Barker, p. 795.

Apparatus Required.—An electro-dynamometer; a copper voltameter; a storage battery of two cells; a reversing-switch; a stop-watch; a rheostat.

Theory of Experiment.—If a current C is sent in series through the fixed and movable coils of a Siemens electro-dynamometer, we have the relation

$$C^2 = K\theta,$$

where θ is the angle through which the torsion index, to which the spiral spring is attached, must be turned to balance the torque due to the current, and K the constant of the instrument. Equation (1) may be evidently written

$$C = K\sqrt{\theta}.$$

If C be measured and θ observed, K can be calculated from formula (2).

The current C may be measured either by a voltameter, a Kelvin balance, or a standard Weston instrument.

We shall assume that a copper voltameter is used.

In this case the value of K is given by the equation

$$K = \frac{m}{t \times .0003285 \times \sqrt{\theta}}$$

where m is the mass of copper deposited, and t the duration of the current in seconds.

Practical Directions.—Set, by means of a compass, the plane of the movable coil of the electro-dynamometer approximately at right angles to the magnetic meridian.

Level the instrument so that this coil swings freely.

Turn the torsion index up against the stop, and, if the instrument be properly adjusted, the needle carried by the coil will remain at zero.

If not, loosen the collar which carries the pointer and adjust the torsion-head until the needle is at zero with the pointer against the stop.

See that the mercury cups at the terminals of the movable coil are full, and that the ends of the coil dipping into them are well amalgamated.

To set the plane of the movable coil accurately at right angles to the meridian, connect the coils in series, through the reversing-switch and rheostat, to the battery, and balance the current.

On reversing the current the needle should return to zero. If not, turn the whole instrument until it does.

Connect the dynamometer, battery, rheostat, and voltameter in series, and adjust the current to 200 degrees approx.

Open the circuit.

Clean, wash, and weigh the copper plate of the voltameter on which the copper deposit is to be made, and replace it in the voltameter.

Close the circuit, taking the exact time.

Allow the current to run for at least 20 minutes, keeping the dynamometer continually balanced by adjusting the contact piece of the rheostat.

Readings should be taken every two minutes to allow for small changes of the current.

A mean of the readings gives the true value of θ.

Open the circuit, taking the exact time.

Wash, dry, and weigh the cathode to $\frac{1}{10}$ of a gram.

Two such determinations should be made with different deflections.

A mean of the constants should be taken for K.

Precautions.—The connections to the dynamometer should be made with thin wire, and the switch and rheostat kept at some distance from it, otherwise the instrument will be affected by the currents in the external circuit.

The dynamometer should also be kept away from strong magnets.

Keep the dynamometer always in an upright position.

Example.—Enter results thus:

SIEMENS DYNAMOMETER No. ....

W	W_1	m	t Minutes.	Torsion Index.	K
				Mean of 15 read'gs	
51.489	53.332	1.844	30	216.47	.213
53.332	54.083	.751	30	33.00	.221
Mean value					.217

Blank to be filled in by student.

W	W_1	m	t Minutes.	Torsion Index.	K
Mean value					

73. TO CALIBRATE AN AMMETER BY A SIEMENS DYNAMOMETER.

References.—As in Experiment 72, and, in addition, S. Thompson, p. 209; Hastings and Beach, p. 241; Nichols and Franklin, vol. II. p. 89.

Apparatus Required.—A Siemens dynamometer; an ammeter of approximately the same range; a rheostat of suitable resistance; a storage battery of several cells.

Theory of Experiment.—Knowing the constant of the dynamometer it can be used very conveniently as a standardizing instrument. If the instrument to be calibrated is of approximately the same range as the dynamometer, it may simply be connected in series with it, and the two instruments read simultaneously at suitable intervals throughout the range. The square root of the reading of the dynamometer, multiplied by its constant, give the correct value of the current for the corresponding indication of the ammeter, which may be either a direct-current or alternating-current instrument.

Practical Directions.—Adjust the dynamometer as in the last experiment.

Set up the ammeter and level it until the needle swings freely and comes to rest at zero.

Connect the coils of the dynamometer in series with the ammeter through the rheostat and battery.

Adjust the current to the initial reading of the ammeter. Usually in the 5-amp.-range instruments this reading is 0.5 amp.

Balance the dynamometer and read both instruments.

Continue to take readings at intervals of 0.50 amp., as indicated by the ammeter.

Calculate the currents corresponding to the indications of the ammeter.

Plot a curve with ammeter readings for abscissas, and the differences between the ammeter reading and the corresponding currents as deduced from the dynamometer readings for ordinates.

Example.—Enter results thus:

Ammeter, Soames & Nalder, range 5 amps.
Siemens Dynamometer, range 4 amps.
Dynamometer constant as previously determined 0.217.

Dyn. Reading.	Am. Reading.	Calculated Current.	Difference.
7.7	.4	.62	+.18
18.0	1.00	0.92	−.08
42.0	1.50	1.40	.10
76.0	2.00	1.89	.11
115.6	2 50	2.33	.17
147.0	2.90	2.64	.26
228.6	3.55	3.28	.27
268.0	3.95	3.58	.27
325.0	4.35	3.91	.44

Blank to be filled in by student.

Dyn. Reading.	Am. Reading.	Calculated Current.	Difference.

Show curve as indicated above.

74. THE D'ARSONVAL GALVANOMETER.—TO DETERMINE ITS RESISTANCE BY SHUNTING IT WITH KNOWN RESISTANCES.

References. — As in Experiment 75 and, in addition, Barker, p. 704; Carhart, pt. II. p. 276; Knott, pt. II. p. 190; Nichols and Franklin, vol. II. p. 56; Watson, p. 694; S. Thompson, p. 409; Hastings and Beach, p. 429; Anthony and Brackett, p. 361.

Apparatus Required.—A D'Arsonval galvanometer, with lamp and scale; a resistance-box of 100,000 ohms; a resistance-box of 2000 ohms; a storage battery or other suitable source of current; a thermometer; a reversing-switch.

Theory of Experiment.—In the D'Arsonval galvanometer the deflection of the galvanometer depends on the strength of the magnetic field in which the coil hangs, the number of windings in the coil, and the current.

Since, for small deflections, the magnetic field in which the coil swings may be considered uniform, the current may be taken as proportional to the deflection, or

$$C = K\delta_1,$$

where C is the current in the coil, and δ_1 the scale deflection.

Suppose a galvanometer G, a large resistance R_1, a battery of E.M.F. E to be connected in series, and the galvanometer shunted by a resistance S_1; then the total current in the circuit is given by the equation

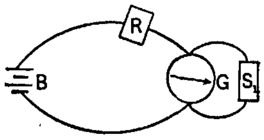

FIG. 50.

$$C = \frac{E}{R_1 + \frac{GS_1}{G + S_1}},$$

and the current flowing through the galvanometer is given by the equation

$$C_1 = \frac{S_1}{G + S_1} \cdot \frac{E}{R_1 + \frac{S_1 G}{G + S_1}} = K\delta_1. \quad . \quad . \quad (1)$$

If now S_1 and R_1 be changed to S_2 and R_2, and the deflection to δ_2, we have again the current in the galvanometer given by the equation

$$C_2 = \frac{S_2}{G + S_2} \cdot \frac{E}{R_2 + \frac{GS_2}{G + S_2}} = K\delta_2. \quad . \quad . \quad (2)$$

In practice it is convenient to make $R_2 = \frac{R_1}{2}$, and to adjust S_2 until the deflection δ_2 is nearly equal to δ_1, so that

$$\delta_2 = \delta_1 + \alpha,$$

where α is a small difference.

If the current be not proportional to the deflection, the last adjustment eliminates any error on that account.

Dividing (1) by (2) and substituting $\delta_1 + \alpha$ for δ_2, and $2R_2$ for R_1, we have

$$1 + \frac{\alpha}{\delta_1} = \frac{S_1 \,.\, 2R_2(G + S_1) + GS_1S_2}{S_1R_2(G + S_2) + GS_1S_2}.$$

Solving for G, and neglecting $\frac{\alpha}{\delta_1} \cdot \frac{S_2}{R_2}$, since α is small as compared with δ_1, and S_2 as compared with R_2, we obtain

$$G = \frac{S_1 S_2}{S_1 - 2S_2}\left(1 - \frac{2\alpha}{\delta_1} \cdot \frac{S_1 - S_2}{S - 2S_2}\right), \quad . \quad . \quad (4)$$

a form convenient for calculation.

Practical Directions.—Set the galvanometer on a bracket or pier free, if possible, from the vibrations of the floors of the room.

Carefully level the instrument until the coil swings freely. The coil of the galvanometer should hang with its plane approximately parallel to the plane of the magnet. Any adjustment for this purpose may be made by means of the torsion head to which the suspension is attached.

Set the lamp and scale about a meter from the galvanometer and obtain an image of the cross-wire on the scale.

If the image be not sufficiently clear, a suitable spectacle-lens should be selected and placed in front of the cross-wire so as to bring it into focus on the scale.

Set the scale at right angles to the line joining its middle point to the mirror, the planes of the mirror and scale being parallel. This can be done by adjusting the position of the scale until its ends are equidistant from the suspension, the image of the cross-wire being at the middle of the scale.

If there is not a thermometer-hole through the case of the instrument, the thermometer should be placed as near the coil as possible.

Connect a two-volt storage battery or a couple of dry cells through a reversing-switch, in series with the galvanometer and a resistance R of from 25,000 to 50,000 ohms.

Shunt the galvanometer with a shunt S_1.

The connections are shown in Figure 51.

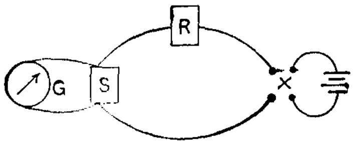

Fig. 51.

Adjust the shunt so as to obtain a deflection of about 300 scale divisions.

Reverse the current and read again. The mean of the readings gives δ_1.

Now, change the resistance in the box R_1 to $\frac{R_1}{2}$ and adjust the shunt S_2 until a deflection as nearly equal to δ_1 as possible is obtained.

Reverse the current and read again.

The mean reading gives δ_2.

$$\alpha = \delta_1 - \delta_2.$$

Repeat the first set of readings again to determine whether changes in the temperature of the resistances used or in the coil or in the E. M. F. of the battery have occurred, and, if a small change has occurred, take the mean of the first and third set of readings for δ_1, keeping R and S unchanged.

Take the temperature of the galvanometer.

The temperature of the shunt should in each case be taken, and corrections made in its resistance.

Example.—Enter results thus:

Nalder galvanometer 3628. Scale distance 103.5 cm.
Shunt-box platinum silver, No. 3750. Correct at 17°.
Temp. coefficient of shunt, 0.00027.

R_1	S_1	δ_1	R_2	S_2	δ_2	Temp. Gal.	Temp. Shunt.	G
50.000	500	301.8 R 301.0 L 302.0 R 301.6 R 301.4 L 303.0 R	25.000	127	302.7 301.6 303.5	19.0	19.2	256.1 at 19°
Mean.......		301.8			302.6	$\alpha = 0.8$		

Blank to be filled in by student.

R_1	S_1	δ_1	R_2	S_2	δ_2	Temp. Gal.	Temp. Shunt.	G
Mean.......						$\alpha =$		

---

75. TO FIND THE CONSTANT, K, OF A D'ARSONVAL GALVANOMETER.

References.—S. Thompson, p. 205; Barker, p. 779; Hastings and Beach, p. 419; Carhart, pt. II. p. 338; Nichols and Franklin, vol. II. p. 46.

Apparatus Required.—A D'Arsonval galvanometer; a tangent galvanometer or copper voltameter: a standard 1-ohm

resistance-coil; a 20,000-ohm box of resistance-coils; a storage-battery; a reversing-switch; three thermometers.

Theory of Experiment.—If a standard cell of constant E.M.F. E be connected in series through a large resistance R, and the galvanometer G, giving a deflection δ, then

$$C = \frac{E}{R + G + B} = K\delta,$$

or

$$K = \frac{E}{(R + G + B)\delta}. \quad . \quad . \quad . \quad . \quad (1)$$

If E, R, G, B be known, K can be calculated.

K can be determined in this way by using a standard Daniell cell and about 25,000-ohm resistance.

For purposes of accuracy, however, the Daniell cell is not sufficiently steady, and the following method is preferable.

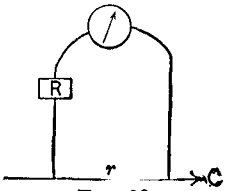

FIG. 52.

If a known current C be passed through a standard resistance r, the terminals of which are connected through a high resistance R to a galvanometer (see Fig. 52), the current c through the galvanometer will be given by the equation

$$c = \frac{Cr}{R + g} = K\delta, \quad \text{or} \quad K = \frac{C\delta}{(R + g)\delta'}, \quad . \quad (2)$$

where g is the resistance, K the constant, and δ the deflection of the galvanometer.

$R + g$ must be large compared with r, and g must be determined beforehand, if not already known.

Practical Directions.—Set up the galvanometer as described in last experiment, and make connections as shown in Fig. 53, *r* being the standard one-ohm coil, and *B* a storage-battery. *G* is the instrument used for measuring the main current.

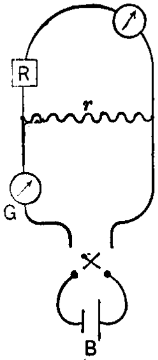

Fig. 53.

If a tangent galvanometer be employed for this purpose, instructions for its use will be found in the experiment on "Absolute Measurement of a Current by Tangent Galvanometer." If a copper voltameter be used, proceed as in previous experiment with copper voltameter. A deflecting instrument for measuring the current is, however, more convenient for this experiment. In either case the method of procedure is as follows:

Adjust the galvanometer current by the resistance-box *R* till a conveniently large deflection, of about 300 scale-divisions, is obtained.

Three galvanometer readings should be taken, the current being reversed each time.

Should a deflecting instrument be used in measuring *C*, it must be read simultaneously with the D'Arsonval galvanometer.

The whole set of readings should be taken twice.

If a copper voltameter is being used, the current should be left running for twenty minutes, and readings of the galvanometer on reversal taken every two minutes, to allow for continual small changes in the current.

Also the reversing-switch must be placed in the galvanometer circuit, and, unless an instantaneous reversing-switch be used, time must be allowed in the calculation of *C* for the reversals.

In either case, a mean of all the galvanometer readings should be taken for δ, and a mean of the main current readings for C.

The temperature of the shunt r, galvanometer g, and resistance R should be noted at the time of observation, and corrections made for them in the calculations if necessary.

The scale distance should be carefully measured and recorded.

Example.—Enter results thus:

D'ARSONVAL GALVANOMETER, NALDER No. 3628.

Data given by makers.—Resistance of galvanometer is 261.95 ohms at 24°.7 C. Resistance r, one-ohm box No. 3660, platinum-silver, correct at 18. Resistance R, platinum-silver box No. 3750, correct at 17°.0 C.

R	t of R	t of r	δ	t of D'Ars. G.	Tangent Galvanometer. $n = 10$.	
					θ	Radius.
11,000	19.00	18.4	340.6 L 333.0 R 338.6 L	17°	47.3 47.4 47.3	18.2
11,000	19.0	18.4	337.5 L 328.0 R 333.8 L		46.7 46.8 46.7	
Mean results, 1st set			336.3		47.33	
Mean results, 2d set			331.8		46.73	

$$C = \frac{10H\,(\text{Radius}) \times \tan\theta}{2\pi n} = .471, \quad \text{1st set,}$$

$$= .466, \quad \text{2d set.}$$

$$K = \frac{.471}{(11,000 + 262)\ 336.3} = \frac{1}{8041000}\ \text{amp.}$$

$$= 1 \text{ volt through } 8.041\Omega, \text{ 1st set.}$$

$$= \frac{466}{(11,000 + 262)\ 331.8} = \frac{1}{8023000}\ \text{amp.}$$

$$= 1 \text{ volt through } 8.023\Omega.$$

Blank to be filled in by student.

R	t of K	t of r_1	δ	t of D'Ars. G.	Tangent Galvanometer. $n =$	
					θ	Radius.
Mean results, 1st set.........						
Mean results, 2d set..........						

$C =$ $\qquad$ $=$

$=$

$K =$ $\qquad$ $=$

$=$

$=$ $\qquad$ $=$

$=$

---

76. TO CALIBRATE THE SCALE OF A D'ARSONVAL GALVANOMETER.

References.—As in previous experiments.

Apparatus Required.—A D'Arsonval galvanometer; two small resistance-boxes; a 10,000-ohm coil; a reversing-switch; a storage battery or Daniell cell; a rheostat; a tangent galvanometer, or a sensitive galvanometer with a potentiometer and Clark cell.

Theory of Experiment.—The deflections of a D'Arsonval galvanometer are not accurately proportional to the current. It is necessary, therefore, to calibrate the scale that is to determine the corrections to be applied for each scale reading.

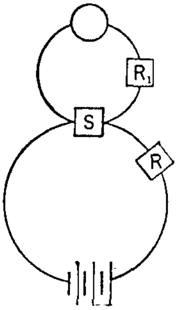

Fig. 54.

Suppose a constant current to be passed through two resistances, R and S, as in Fig. 54, S being connected in series with a galvanometer and a large resistance R_1.

Suppose S so small as compared with R_1 that any small variation of S will not materially alter the current, so long as $R + S$ is constant. Then the difference of potential on the galvanometer terminals will be proportional to the resistance S. The deflections of the galvanometer will therefore also be proportional to S, assuming that the deflection is proportional to the current.

If the deflections for different values of S be observed, their differences from proportionality can be calculated and a correction curve plotted.

Practical Directions.—Connect the galvanometer, battery, and resistances as in Fig. 55, putting the reversing-switch in the galvanometer circuit only.

If the source of current used be not as constant as required, it can be kept constant by placing in the battery circuit a rheostat and tangent galvanometer, and adjusting continuously the rheostat so as to keep the deflection of the tangent galvanometer constant. A better method still would be to connect the terminals of the battery to a high-resistance potentiometer and balance it continuously against a Clark cell. This can be done by having the resistance in the Clark-cell circuit constant and adjusting the rheostat.

The connections in this case would be as in Fig. 55.

If R_1 be 10,000 ohms and the resistance S varies from 1 to 10 ohms, $R + S$ being kept constant, the main circuit resistance will vary less than .01 ohm due to the change in S and the difference of potential on the galvanometer terminals

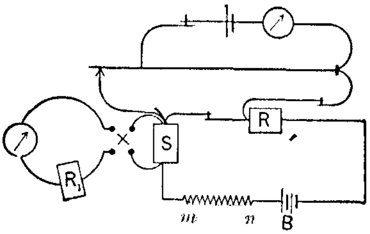

FIG. 55.

will vary less than $\frac{1}{2000}$ from proportionality as a result of this change.

The work of the rheostat will therefore be largely to compensate for changes in the E.M.F. of the battery.

Unplug 10,000 ohms from box R_1 in galvanometer circuit, and 10 from box S.

Unplug from the box R a resistance until a suitable deflection of say 350 scale divisions is obtained.

If the constant K has already been found, as in previous experiment, then the resistance R may be adjusted until the same deflection is obtained as that for which K was calculated, and the differences from proportionality calculated with regard to it.

Having obtained a suitable deflection, reverse the current and mean the readings, to eliminate errors due to torsion. Now plug in one ohm in S and unplug one in R, and adjust the rheostat *mn* for a balance against the Clark cell.

Read again and reverse.

Continue the process right down the scale.

Calculate what the deflection should be in each case, and

plot a curve with scale-readings as abscissas and differences as ordinates.

The calculated deflections will be obtained in each case by taking nine-tenths, eight-tenths, seven-tenths, etc., of the first deflection.

Example.—Enter results thus:

D'Arsonval Galvanometer, No. 3628.

S	Mean Deflection.	Calculated Deflection.	Differences.
10.......	348.25	345.5	2.75
9.......	313.2	311.0	2.2
8.......	278.2	276.4	1.8
7.......	243.0	241.9	1.4
6.......	208.2	206.3	.9
5.......	173.2	172.8	.4
4	138.5	138.2	.3
3.......	104.1	103.6	.5
2.......	69.5	69.1	.4
1.......	34.9	34.5	.4
0.......			
1.......	35	34.5	.5
2.......	69.5	69.1	.4
3.......	104.1	103.6	.5
4.......	138.5	138.2	.3
5.......	172.7	172.8	− .1
6.......	206.8	207.3	− .5
7.......	240.8	241.9	− 1.1
8.......	275.1	276.4	− 1.3
9.......	308.5	311	− 2.5
10.......	342.7	345.5	− 2.8

Blank to be filled in by student.

S	Mean Deflection.	Calculated Deflection.	Differences.

Plot curve as directed.

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77. TO MEASURE POTENTIAL DIFFERENCES BY A D'ARSONVAL GALVANOMETER.

References.—As in Experiments 74 and 75.

Apparatus Required.—A galvanometer; a 100,000-ohm resistance-box; two 10,000-ohm boxes; a reversing-key; a number of batteries.

Theory of Experiment.—The scale of a D'Arsonval galvanometer having been calibrated, the values of G and K determined, it may be used for the measurement of potential differences. With little variation of method, potential differ-

ences varying from a few micro-volts to the volts or the lighting circuit may be determined.

I. The measurements may be made by connecting a large resistance in series with the galvanometer and the source of current, in which case

$$E = K\delta(R + G), \quad . \quad . \quad . \quad . \quad . \quad (1)$$

the terms having the same meaning as previous experiments, where K, R, G are all known quantities and δ is observed.

If the volts on the lighting circuit be determined by this method, R would require to be a resistance of several megohms. In the case of batteries R will be so large that the battery resistance can be neglected.

II. Let the source of current, B, be connected to a large resistance, R, and the galvanometer terminal to two potential points, A, C, of this resistance, with a resistance r between them (Fig. 56).

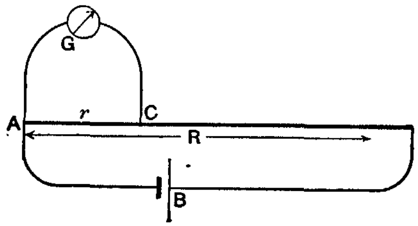

FIG. 56.

Then the current through the galvanometer is given by the equation

$$C = \frac{E}{R - r + \frac{rG}{r + G}} \times \frac{r}{r + G} = K\delta,$$

or $$E = K\delta \frac{\{(R - r)(r + G) + rG\}}{r},$$

$$E = K\delta \left\{ \frac{R(r + G)}{r} - r \right\} . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In practice r is very small in comparison with $\frac{R(r + G)}{r}$ and may therefore be neglected.

Hence $$E = \frac{K\delta R(r + G)}{r} . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

• III. A variation of this method would be to put a resistance R_1 in series with the galvanometer. This would necessitate making R smaller and r larger. In this case we would have the equation

$$C = \frac{E}{R - r + \frac{r(R_1 + G)}{R_1 + G + r}} \times \frac{r}{R_1 + G + r} = K\delta$$

for the current through the galvanometer.

Solving for E, we obtain

$$E = \left\{ \frac{R(R_1 + G + r)}{r} - r \right\} K\delta, \quad . \quad . \quad . \quad (4)$$

or, neglecting r as compared with $\frac{R(R_1 + G + r)}{r}$,

$$E = \frac{R(R_1 + G + r)}{r} K\delta . \quad . \quad . \quad . \quad . \quad (5)$$

Equation (5) may be used in calculating the volts on the lighting circuit, equation (4) in the case of batteries.

Practical Directions.—I. If sufficiently large resistances are available to make it possible to obtain readings directly, connect in series the source of E.M.F., the galvanometer, and the resistances.

In case of batteries 40,000 to 50,000 ohms will be necessary; in case of the lighting circuit a resistance from 2 to 3 megohms will be required.

The reversing-key should be in the galvanometer circuit. Do not close the key until you are *absolutely certain* that the connections are correct and the resistances are all unplugged, otherwise damage may be done the resistance-boxes or the galvanometer.

Read the deflection. Reverse the current and read again. Correct the readings from the calibration curve in each case, and take the mean for δ.

Measure the E.M.F. of each of the batteries given.

If the lighting circuit be direct-current, measure its voltage.

II. Connect the source of E.M.F. to 100,000 ohms resistance, through a key, which must be left open.

Unplug the 100,000 ohms from the box if the lighting circuit is to be determined.

Connect the galvanometer terminals to the potential-points by means of a travelling plug on the box.

If the box does not contain a travelling plug, it will be necessary to put in a small resistance-box in series with the 100,000 ohms and use it for adjusting r. In this case R will be equal to $100,000 + r$.

Adjust r until a suitable deflection is obtained.

For a storage-battery R will be about 5000 ohms and r about 20 ohms.

Repeat the observations as in I.

III. Now put 10,000 ohms, R_1, in series with G, and reduce R to 10,000 and adjust as before.

Make a diagram of the connections, and be sure you understand the connections before proceeding.

The value of r in this case will be about 50 ohms for the lighting circuit and about 2500 for a storage-battery.

Repeat the observations as in I and II.

In the values suggested for r, R, and R_1 above the galvanometer, of which $K = \frac{1}{8.03}\Omega$ and $G = 261.95$ ohms at 24.7° C., is referred to. The student can easily determine beforehand approximately the corresponding values for any other instrument once its constants are known.

Precautions.—The lighting voltage must not be applied to an accurately adjusted and valuable resistance-box without at least 10,000 ohms being unplugged. Be sure that the connections are correct before closing the circuit.

Example.—Enter results thus:

$K = \frac{1}{8,030,000}$. G (corrected for temperature) = 256.

Method	Source of E.M.F.	R	R_1	r	Corrected δ	E.M.F.
I.	Lighting circuit...	2500000			316.5	98.5
II.	"	100000		10	295.5	97.9
III.	"	10000	10000	50	365.4	98.2
I.	Storage-battery....	50000			354.7	2.22
II.	"	5000		20	262.0	2.25
III.	"	10000	10000	2500	369	2.28
I.	Leclanché battery	40000			259.4	1.30
II.	"	5000		30	219.0	1.29
III.	"	10000	10000	2500	215.2	1.30

Blank to be filled in by student.

$K =$ $G =$

Method.	Source of E.M.F.	R	R_1	r	Corrected δ	E.M.F.
I.						
II.						
III.						
I.						
II.						
III.						
I.						
II.						
III.						

78. TO CALIBRATE A MILLI-VOLT METER.

References.—S. Thompson, p. 208; Barker, p. 720; Hastings and Beach, p. 423.

Apparatus Required.—A milli-volt meter, two single-cell storage-batteries; two resistance-boxes, one with an ohm divided to tenths; a potentiometer; a sensitive galvanometer.

Theory of Experiment.—The calibration of instruments for measuring currents or potential differences can be effected by means of the calibrated D'Arsonval galvanometer used in the previous experiments by shunting the galvanometer. The following method, however, is simpler and the results are more easily calculated.

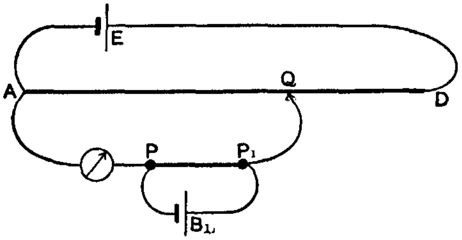

FIG. 57.

Suppose P and P_1 the terminals of milli-volt meter, across which a current is flowing from the battery B_1, to be connected through a galvanometer to a potentiometer (Fig. 57), upon the terminals of which is a constant E.M.F. E; then if the sliding contact Q be adjusted until no deflection of the galvanometer is obtained, we have the relation

$$\frac{V}{E} = \frac{\text{Resistance } AQ}{\text{Resistance } AD},$$

or

$$V = \frac{\text{Resistance } AQ}{\text{Resistance } AD} \times E, \quad . \quad . \quad . \quad . \quad (1)$$

where V is the potential difference between P and P_1, and E that of the constant battery.

If, therefore, the indications of the instrument corresponding to different values of V be observed, and these indications be compared with the calculated values, an absolute calibration curve for the instrument can be drawn.

The object of the present experiment is to find a correction curve for an instrument which has already been calibrated.

Practical Directions.—In practice it is necessary to have a standard Clark or Weston cell in the system as well as the constant battery through the potentiometer. For connections see Fig. 58.

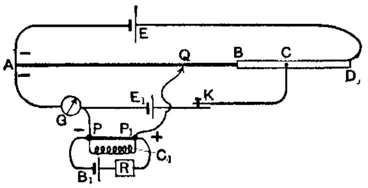

FIG. 58.

The total resistance AD consists of a stretched wire AB, with sliding contact at Q, and a resistance-box BD, with the small resistances toward the end D.

E_1 is a standard cell connected through the key K to a fixed point C by means of a travelling plug on the box.

P and P_1 are the terminals of the milli-volt meter, B_1 the battery producing its deflections, R a resistance in the circuit for varying the deflections, and C_1 the coil of the instrument.

E and E_1 are kept continuously balanced against each other, so that the milli-volt meter is calibrated against the standard cell. Hence the resistance AC must be substituted for AD in (1).

The resistances in the various parts of the system will depend on the instrument to be calibrated. It is convenient to adjust the resistance between B and C so that a milli-volt corresponds to a definite length of the wire AB.

For example, if the instrument to be calibrated be a Weston milli-volt meter with a range of 10 milli-volts, the wire AB may conveniently be a B. A. bridge wire of approximately one ohm resistance, and BC adjusted so that a milli-volt corresponds to ten centimetres of the bridge wire.

The value of the resistance to be unplugged between B and C in this case is at once determined from the relation

$$\frac{.001}{1.434} = \frac{.1 \times r}{x + r},$$

where 1.434 is the E.M.F. of the standard cell, r the resistance of the whole wire AB, assumed to be a metre in length, and x the unknown resistance.

Hence $$x = 142.4 \times r.$$

If, therefore, between B and C 142 or 143 ohms be unplugged, each milli-volt will approximately correspond to ten centimetres of the wire, and the reading can be taken if necessary to $\frac{1}{100}$ of a milli-volt.

Hence, measure the resistance of AB.

Calculate the approximate value of x.

Unplug resistances between C and D until on closing K no deflection of the galvanometer is obtained.

For E and B_1 single-cell storage-batteries are suitable.

Adjust R until a deflection of one milli-volt is obtained, denoting the deflection by δ.

R can usually be adjusted to give exact milli-volt readings.

Adjust the sliding contact, Q, until no deflection of the galvanometer is obtained.

Test the balance of E_1 and E before and after the observation, adjusting always between C and D.

Record the position of Q, and the reading on the instrument.

Repeat the observations for a number of points up the scale.

If a Clark cell be used, take its temperature and correct its E.M.F. from the equation

$$E_1 = 1.434 - .0012(t^\circ - 15), \quad . \quad . \quad . \quad (2)$$

Calculate the value V corresponding to each reading from equation

$$V = \frac{\{1.434 - .0012(t^\circ - 15)\} AQ \times r}{(r + x)100}, \quad . \quad (3)$$

Precautions.—Do not short-circuit the standard cell.

Before connecting R to the battery and milli-volt meter, unplug at least 100 ohms.

Connect the negative poles of the batteries to the same end of the potentiometer wire.

Example.—Enter results thus:

WESTON MILLI-VOLT METER. 10-MILLI-VOLT RANGE.

Temp. of Clark cell 16.5. $E_t = 1.432$. $r = .90$ ohms.

ΔQ	δ	v milli-volts.	$\delta - v$
10.	1.00	.915	.085
21.	2.00	1.93	.07
31.6	3.00	2.89	.11
42.0	4.00	3.84	.16
52.3	5.00	4.78	.22
63.2	6 00	5.78	.22
73.9	7.00	6.76	.24
84.5	8.00	7.73	.27
95.6	9 10	8.75	.25

Blank to be filled in by student.

ΔQ	δ	v milli-volts.	$\delta - v$

Plot a curve for δ and $\delta - v$.

79. TO DETERMINE THE LOGARITHMIC DECREMENT OF A BALLISTIC GALVANOMETER.

References.—Hastings and Beach, p. 420; Barker, p. 780; S. Thompson, p. 207.

Apparatus Required.—A ballistic galvanometer; a damping-coil; a battery; a contact-key.

Theory of Experiment.—The ballistic galvanometer is an instrument for measuring currents of very short duration. The needle is long and heavy, so that its time of vibration is very large, the time of the passage of the transient current being assumed so short that the needle remains at rest during its passage.

In making measurements depending on such currents the swing of the needle and not the permanent deflection is observed, and hence it is necessary to consider how much the amplitude of the vibration of the needle is affected by the damping due to resistance of the air and other causes. In a ballistic galvanometer, the needle being heavy, the damping is usually very small.

It may be demonstrated mathematically or shown experimentally that the effect of damping is to diminish the amplitudes of the successive vibrations in a fixed proportion. Thus if a_1, a_2, a_3, . . . a_n be the successive amplitudes of vibration of a needle, then

$$\frac{a_1}{a_2} = \frac{a_2}{a_3} = \frac{a_{n-1}}{a_n} = r,$$

and hence $$\frac{a_1}{a_n} = r^{n-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Hence $$\log_e a_1 - \log_e a_n = (n - 1) \log_e r,$$

or $$\log_e a_1 - \log_e a_n = (n - 1)\lambda,$$

where $$\lambda = \log_e r.$$

Hence $$\lambda = \frac{1}{n - 1} (\log_e a_1 - \log_e a_n). \quad . \quad . \quad . \quad (2)$$

The value λ is called the *logarithmic decrement.*

We will now show that the effect of damping on the amplitude of any swing is to diminish it by $\frac{\lambda}{2}$.

Suppose the galvanometer needle to be set swinging and the amplitude of the first swing to be l.

This amplitude is shorter than the true amplitude, since the needle has been damped through a half swing.

Denoting by l_0 the true swing, that is, the swing that would have been observed had no damping been present, then, from (2),

$$\lambda = \frac{1}{\frac{1}{2}}(\log_e l_0 - \log_e l),$$

and hence $$\tfrac{1}{2}\lambda = \log_e l_0 - \log_e l,$$

or $$\log_e l_0 = \tfrac{1}{2}\lambda + \log_e l.$$

Hence $l_0 = e^{\frac{1}{2}\lambda + \log_e l}$

$$= e^{\frac{\lambda}{2}} \times e^{\log_e l}$$

$$= e^{\frac{\lambda}{2}} . \, l$$

$$= l\left(1 + \frac{\lambda}{2} + \frac{1}{2}\left(\frac{\lambda}{2}\right)^2 + \text{etc.}\right)$$

$$= l\left(1 + \frac{\lambda}{2}\right), \text{ if the damping be small.}$$

Hence if the observed first swing of a galvanometer be l, the true swing is given by the equation

$$l_0 = l\left(1 + \frac{\lambda}{2}\right). \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Practical Directions.—Set the galvanometer so that the needle swings freely, and adjust the lamp and scale until the spot of light is in focus on the scale.

Connect the damping-coil and battery through the contact-key, and place the coil close to the coil of the galvanometer. By tapping the key the action of the current in the damping-coil will set the galvanometer-needle swinging. A little practice with this coil will enable the student to bring the swinging needle quickly to rest.

Set the needle swinging through about 300 scale-divisions, and observe the turning-point of the spot of light on the scale, following it as it swings, and again reading its turning-point on the opposite side of the scale.

Count from 30 to 50 complete vibrations, taking again the turning-point at the beginning and end of the last swing.

The sum of the first two readings gives the first swing; the sum of the last two readings gives the last swing.

Calculate λ from equation (2), multiplying the common logarithms by 2.3026, the modulus of reduction to the base e.

Repeat the observations, taking mean value for λ.

Now, without altering the control magnets, take carefully the time of 20 complete vibrations and thus obtain T, the periodic time of the needle.

Alter the sensitiveness of the galvanometer by changing the position of the control magnets, and repeat the observations for λ and T.

Repeat the whole set of observations several times, changing the position of the control magnets each time.

Plot a curve with values of T for abscissas and the corresponding values of λ for ordinates.

The value of λ for any given sensitiveness as defined by the periodic time can now be obtained from the curve.

Example.—Enter results thus:

First Swing.	Last Swing.	Number of Swings.	T	λ
352.5	164.5	50	6.3	.0155
328.5	170.5	50	5.3	.0134
351.0	210.0	50	4.3	.0108
347.0	217.0	50	3.9	.0096
320.0	220.0	50	3.3	.00765

Blank to be filled in by student.

First Swing.	Last Swing.	Number of Swings.	T	λ

80. TO DETERMINE THE ABSOLUTE CAPACITY OF A CONDENSER BY A BALLISTIC GALVANOMETER.

References.—S. Thompson, p. 425; Barker, p. 562; Ames, pp. 294–303; Carhart, pt. II. pp. 201–210; Anthony and Brackett, pp. 291–295; Nichols and Franklin, pp. 65–67; Hastings and Beach, p. 339; Watson, p. 643; Knott, pt. II. p. 136.

Apparatus Required.—A ballistic galvanometer; a condenser the capacity of which is to be determined; a resistance-box for shunting the galvanometer; a large resistance; several batteries; one tapping contact-key; three contact-keys that can be permanently closed.

Theory of Experiment.—The capacity of a condenser is the ratio of the charge required to produce a certain difference of potential between its plates to the potential.

If C be the capacity, Q the charge, and V the difference of potential between the plates,

$$C = \frac{Q}{V}.$$

Suppose the condenser to be charged with a potential V through a ballistic galvanometer, in which case all the charge may be considered as having passed through the coils before the needle began to move.

Then if G be the galvanometer constant, M the magnetic moment of the magnet, the total impulse on the needle is

$$MGQ.$$

If ω be the angular velocity with which the needle begins

to move and I be its moment of inertia, then $I\omega$, the moment of momentum, is equal to the impulse communicated by the charge.

Hence $$I\omega = MGQ. \quad \ldots \ldots \quad (1)$$

Now suppose the original position of the needle to be AB, and CD the position at the end of a swing, α being

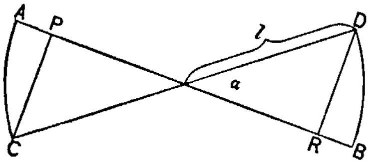

FIG. 59.

the angle through which the needle swings. Then the total displacement of the north pole is AP, and of the south pole BR, and the work done against the earth's magnetic field to produce this displacement is given by the equation

$$W = Hm(AP + RB),$$

where H is the earth's horizontal component and m the strength of one pole. Hence

$$W = 2Hml(1 - \cos\alpha)$$

$$= HM(1 - \cos\alpha)$$

$$= 2HM\sin^2\frac{\alpha}{2}. \quad \ldots \ldots \quad (2)$$

But the work done is also equal to the kinetic energy of the needle.

Hence $$\frac{I\omega^2}{2} = 2MH\sin^2\frac{\alpha}{2}. \quad \ldots \ldots \quad (3)$$

Equating the values of ω found from (1) and (3) and solving for Q, we obtain

$$Q = \frac{2 \sin \frac{\alpha}{2}}{G} \times \sqrt{\frac{HI}{M}}. \quad . \quad . \quad . \quad . \quad (4)$$

If T be the time of a complete oscillation of the needle,

$$MH = \frac{4\pi^2 I}{T^2},$$

and therefore $$\frac{I}{M} = \frac{H.T^2}{4\pi^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Substituting in (4),

$$Q = \frac{TH}{\pi G} \sin \frac{\alpha}{2},$$

or, since $\frac{H}{G} = K$, the reduction factor of the galvanometer,

$$Q = \frac{KT \sin \frac{\alpha}{2}}{\pi}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Hence $$C = \frac{Q}{V} = \frac{KT \sin \frac{\alpha}{2}}{\pi V}. \quad . \quad . \quad . \quad . \quad . \quad (7)$$

If now the same potential difference be connected to the galvanometer terminals through a resistance R, the galva-

nometer being shunted with a resistance S, the current through the galvanometer is given by the equation

$$\gamma = \frac{V}{R + \frac{GS}{G+S}} \times \frac{S}{G+S} = K \tan\theta,$$

or

$$\frac{VS}{R(G+S)+GS} = K\tan\theta,$$

and hence

$$\frac{K}{V} = \frac{S}{\{R(G+S)+GS\}\tan\theta}. \quad . \quad . \quad (8)$$

Substituting in (7),

$$C = \frac{TS\sin\frac{1}{2}\alpha}{\pi\{(G+S)R+GS\}\tan\theta}. \quad . \quad . \quad (9)$$

In the above we have supposed that no damping was present when the needle was displaced by the charge, and hence for sin $\frac{1}{2}\alpha$ we must write

$$\left(1+\frac{\lambda}{2}\right)\sin\tfrac{1}{2}\alpha,$$

and (9) becomes

$$C = \frac{TS\left(1+\frac{\lambda}{2}\right)\sin\frac{1}{2}\alpha}{\pi\{(G+S)R+GS\}\tan\theta}. \quad . \quad . \quad (10)$$

All the quantities on the right-hand side of (10) can be observed and hence C determined.

In this and other experiments on condensers the observations are taken when the condenser is charged through the galvanometer, thus obtaining the *instantaneous capacity*. The value obtained will usually differ from that obtained on discharge, the difference being due to absorption.

Practical Directions.—A simple and convenient way of connecting the apparatus so as to enable the observer to take the two sets of observations in rapid succession is shown in Fig. 60.

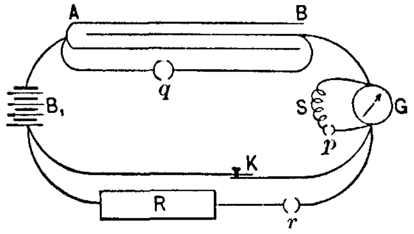

FIG. 60.

AB is the condenser, B_1 the source of E.M.F., *K* a tapping contact-key, *q*, *r*, and *p* plug contact-keys, *S* the shunt, and *R* a large resistance. If the condenser be provided with a discharging-plug, as is usual, *q* will not be necessary. If *q*, *r*, and *p* be left open and *K* closed with a quick tap, the condenser will be charged through the galvanometer.

Closing *q* discharges the condenser, closing *p* shunts the galvanometer, closing *r* brings in the large resistance *R*, and the observations for the steady current can be made.

A few preliminary trials will first be necessary to determine the number of batteries to be used to give a suitable throw of the needle. *R* and *S* should also be adjusted in a preliminary trial to obtain a suitable deflection.

All the connecting wires should be carefully insulated to prevent leakage.

Bring the needle to rest by means of the damping-coil.

Close *K* with a sudden tap, freeing it as quickly as possible, and observe the throw of the needle.

Repeat the observations several times, taking the mean throw.

Close the keys p, q, r, and observe the deflection.

Read R, S, and δ.

Calculate $\sin \frac{1}{2}\alpha$, knowing that $\tan 2\alpha = \frac{\text{throw}}{\text{scale distance}}$.

Calculate $\tan \theta$, knowing that $\tan 2\theta = \frac{\delta}{\text{scale distance}}$.

If the throw and deflection be both small and the scale distance of the galvanometer large, it will usually be sufficient to substitute for $\sin \frac{1}{2}\alpha$ one-half the throw of the needle, and for $\tan \theta$ the deflection.

Repeat the observations.

Take the time of 50 swings of the needle, and calculate T, the time of a complete oscillation.

λ can be obtained from the curve for the logarithmic decrement by means of T if the galvanometer be the one used in the last experiment, otherwise λ must be obtained directly.

In the example given below 6 storage-cells were used.

Example.—Enter results thus:

NALDER ⅓ MICRO-FARAD No. 3471.

Throw.	R	S	δ	T	$\frac{\lambda}{2}$	C
104 106	1Ω "	1000 "	184.7 185.0	8″.0	.006	.332

Blank to be filled in by student.

Throw.	R	S	δ	T	$\frac{\lambda}{2}$	C

81. TO COMPARE THE CAPACITIES OF TWO CONDENSERS. DIRECT-DEFLECTION METHOD.

References.—As in last experiment.

Apparatus Required.—A condenser whose capacity has been determined; condensers for comparison; a ballistic galvanometer; several batteries; a Pohl commutator; a contact-key.

Theory of Experiment.—If a condenser be charged to a potential v through a ballistic galvanometer, we have from equation (7) of the last experiment the relation

$$C = \frac{KT}{\pi v} \sin \frac{\alpha}{2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

the terms having the same meaning as in that case.

Similarly if a second condenser be charged with the same potential,

$$C_1 = \frac{KT}{\pi v} \sin \frac{\alpha}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence

$$\frac{C_1}{C} = \frac{\sin \frac{\alpha_1}{2}}{\sin \frac{\alpha}{2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If observations for α and α_1 be made, C_1 can be calculated if C be known.

Practical Directions.—A convenient method of making the connections is shown in Fig. 61.

AB and *CD* are the two condensers, *abcdef* a Pohl commutator with the connectors on the base removed, B_1 the source of E.M.F., K_1 a contact-key.

The battery and galvanometer are connected to the contacts in which the rocker dips. On rocking the com-

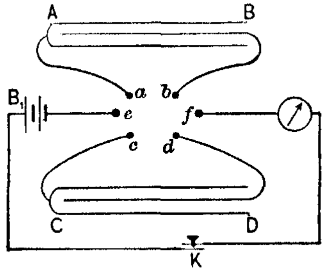

FIG. 61.

mutator toward *AB*, *ae* and *bf* are connected, and on closing *K* the condenser *AB* is charged through the galvanometer. On rocking toward *CD* the same thing occurs for the condenser *CD*.

The observations can then be taken in rapid succession, if the galvanometer needle be brought to rest with a damping-coil.

The condensers must be discharged after each observation.

If they are not supplied with plugs for the purpose, they can be short-circuited through the points in which they make contact with the Pohl commutator. If the battery be connected directly through *K* to *f*, and the galvanometer to the rocker terminals of another Pohl commutator with base connectors removed, the other terminals being connected in pairs to *ab* and *cd*, then the condensers can be charged by rocking the first and closing *K*, and discharged through the gal-

vanometer by rocking the second commutator, thus giving a comparison on discharge. Compare the several condensers given with the standard.

If the galvanometer-throws be nearly equal, they may be substituted for $\sin \frac{\alpha}{2}$ and $\sin \frac{\alpha_1}{2}$ in equation (3), otherwise $\sin \frac{\alpha}{2}$ and $\sin \frac{\alpha_1}{2}$ must be calculated.

Example.—Enter results thus:

Condenser.	Throw.	Scale Distance.	$\sin\frac{a}{2}$	C
Standard $\frac{1}{3}$ M. F.......	225	1 Meter		.332
.2 M. F......	142.3	"		.210
.5 M. F......	325	"		.495

Blank to be filled in by student.

Condenser.	Throw.	Scale Distance.	$\sin\frac{a}{2}$	C

82. TO COMPARE THE CAPACITY OF CONDENSERS. METHOD OF MIXTURES.

References.—As in Experiment 79.

Apparatus Required.—A sensitive reflecting galvanometer; two resistance-boxes; two Pohl commutators; a contact-key; several batteries.

Theory of Experiment.—In this method the condensers are charged so that the two charges are equal, the potentials producing them being unequal.

Since $C = \frac{Q}{V}$ and $C_1 = \frac{Q}{V_1}$,

then $$C_1 = \frac{VC}{V_1}. \qquad (1)$$

If the condensers be charged and the charges allowed to mix in such a way as to neutralize each other, the system being then discharged through a galvanometer, the charges are equal if no deflection is obtained.

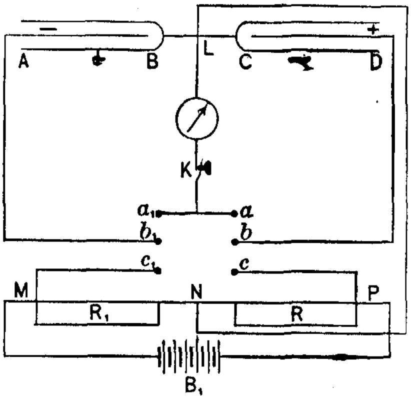

FIG. 62.

Practical Directions.—The connections can be made as in Fig. 62.

AB and CD are the two condensers, R and R_1 two resistance-boxes connected to the terminals of a battery B_1, a, b, c, a_1, b_1, c_1, the terminals of a Pohl commutator from which the base connectors have been removed.

By rocking the switch so as to connect b_1 to c_1, and b to c, the two condensers will be charged with potentials proportional to R and R_1, so that

$$\frac{R}{R_1} = \frac{V}{V_1}.$$

By rocking the switch so as to connect a_1 to b_1, and a to b, the battery is thrown out of the circuit and the charges on the two condensers neutralize each other, the negative plates of the one being connected to the positive plates of the other.

If the charges are equal, a complete neutralization takes place. If not, the two make one condenser system charged with the difference between the two charges, and on closing K a discharge will take place through the galvanometer.

If an approximate relation between C and C_1 be known, R and R_1 can be roughly adjusted. Otherwise their values can only be determined by trial.

Repeat the adjustment until no deflection is obtained.

Between the trials the condensers should be thoroughly discharged. This can be done by keeping K closed for a few seconds after each discharge.

Record the values of R and R_1.

Compare the condensers given with the standard, and calculate their values in each case.

Example.—Enter results thus:

Condenser.	R	R_1	C	C_1
Standard ⅓ M. F......	2000		.332	
.2 M. F......		3400		.195 M. F.
.5 M. F......		1295		.490

Blank to be filled in by student.

Condenser.	R	R_1	C	C_1

83. TO COMPARE THE CAPACITIES OF CONDENSERS. BRIDGE METHOD.

References.—As in Experiment 79.

Apparatus Required.—As in the last experiment.

Theory of Experiment.—In the last experiment the charges were equal and the potentials unequal; in this experiment the potentials are equal and the charges unequal. Suppose the

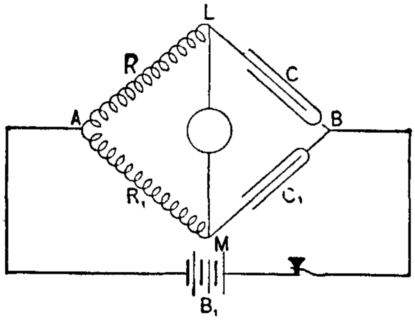

FIG. 63.

two condensers and the two resistances connected in the arms of a Wheatstone bridge, Fig 63, C and C_1 being the condensers, R and R_1 the two resistances.

In order that no current flow through the galvanometer on charging the condensers, that is, on closing K, L and M must be at the same potential.

Denote the common potential of L and M by V and the potential of A by V_1.

The total quantity of electricity which flows into C is therefore give by the relation

$$Q = \gamma t = \left(\frac{V_1 - V}{R}\right)t, \quad . \quad . \quad . \quad . \quad (1)$$

where γ is the current and t the short time required to charge the condenser.

Similarly the relation for C_1 is

$$Q_1 = \gamma_1 t = \left(\frac{V_1 - V}{R_1}\right) t. \quad . \quad . \quad . \quad . \quad (2)$$

Hence, from (1) and (2),

$$\frac{Q}{Q_1} = \frac{R_1}{R}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

But $$C = \frac{Q}{V}, \text{ and } C_1 = \frac{Q_1}{V},$$

the potentials being the same on the plates.

Hence $$\frac{C}{C_1} = \frac{Q}{Q_1} = \frac{R_1}{R},$$

or $$C_1 = \frac{RC}{R_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If, therefore, R and R_1 be adjusted so that on charging the condensers no deflection of the galvanometer is obtained, C_1 can be calculated from (4), C being known.

Practical Directions.—The connections can be made exactly as in Fig. 63.

Be careful to insulate all the parts of the apparatus.

Adjust R and R_1 until no deflection is obtained on closing K.

Between the trials the condensers must be discharged completely.

R and R_1 should be as large as possible.

A ballistic galvanometer is not necessary; any sensitive galvanometer will serve the purpose.

Compare the condensers as in previous experiments.

Example.—Enter results thus:

Condenser.	R	R_1	C	C_1
Standard $\frac{1}{3}$ M. F.......	2000		.332	
.2 M. F.......		3400		.195
.5 M. F.......		1295		.490

Blank to be filled in by student.

Condenser.	R	R_1	C	C_1

84. MEASUREMENT OF E.M.F. AND BATTERY RESISTANCE BY CONDENSER METHOD.

References.—S. Thompson, p. 422.

Apparatus Required.—A condenser; a ballistic galvanometer; a resistance-box; a contact-key; batteries for measurement.

Theory of Experiment.—(1) If a condenser of capacity C be charged, by means of a battery, with E.M.F. E, we have the relation (see (7) page 271)

$$CE = K_1 \sin \tfrac{1}{2}\alpha. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If the same condenser be charged with an E.M.F. E_1,

$$CE_1 = K_1 \sin \tfrac{1}{2}\alpha_1. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence
$$\frac{E}{E_1} = \frac{\sin \frac{1}{2}\alpha}{\sin \frac{1}{2}\alpha_1} = \frac{\delta}{\delta_1}. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

approximately, where δ and δ_1 are the galvanometer throws in the two cases.

With a standard condenser the method is suitable for comparing the electromotive forces of batteries.

(2) We have for a battery charging the condenser the relation

$$CE = K\delta.$$

If now the battery be shunted with a shunt S and the condenser charged, we have the relation

$$CE \times \frac{S}{B + S} = K\delta_1.$$

Hence
$$\frac{B + S}{S} = \frac{\delta}{\delta_1},$$

or
$$B = \frac{S(\delta - \delta_1)}{\delta_1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

from which the battery resistance B can be calculated.

Practical Directions.—(1) Connect in series a Clark cell, the galvanometer, and the condenser through a contact-key. The Clark cell should be connected in by means of a three-way key in addition to the contact key, thus allowing it to remain

permanently in the connections, while another battery to be compared with it can be also connected and each used in turn by simply altering the plug in the three-way key. Observe the throw for each of the two batteries in succession, δ and δ_1, and also the throw for each shunted.

Close the shunt through a contact-key pressing the key only for the moment necessary to make the observation.

Special care must be taken not to short-circuit the Clark cell, as it polarizes very rapidly.

Read the temperature, t, of the Clark cell.

Assuming the E.M.F. of the Clark cell to be

$$1.434 - .0012(t - 15),$$

calculate the electromotive forces of the others.

Calculate their resistances from equation (4).

Example.—Enter results thus:

Temperature of C. C. 16.5

Battery.	δ	S	δ_2	E	B
Clark cell..........	65	80	40	1.432	50.
Leclanché..........	61	10	50	1.35	2.2
Storage-battery.....	105	10	97	2.31	0.8
Daniell............	49	50	45	1.07	4.5

Blank to be filled in by student.

Battery.	δ	S	δ_2	E	B

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